

# From ‘Questions about Compositionality’

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## 8 Quotation: a counter-example?

I will not discuss here the many counter-examples to compositionality that have been proposed, and the compositional solutions that have been suggested. But I will look at one case, which is perhaps the clearest of them all: (pure) quotation, i.e. the ability to refer in the language to linguistic expressions (meaningful or not). In a perfectly clear, and in principle familiar, sense, quotation is not compositional. Let us make this a bit more precise.

A language  $L$  is, as above, identified with a constituent structure  $(\mathbb{E}, \mathbb{F})$  with a distinguished cofinal set  $X \subseteq \mathbb{E}$  of (declarative) sentences, and a semantics  $\mu$  with domain  $X$ . We say that  $L$  is *interpreted* if each sentence is either true or false, and that  $\mu$  *respects truth values* if whenever  $e$  and  $f$  differ in truth value,  $\mu(e) \neq \mu(f)$ .

I will further say that  $L$  *has quotation* if there is a unary frame  $Q \in \mathbb{F}$  such that, intuitively,  $Q(e)$  is a quote frame of  $e$  (e.g.  $e$  surrounded by quotation marks) when  $e \in X$ ,<sup>1</sup> and  $L$  is able to express elementary syntactic properties of sentences. The details need not be specified, but the point is there are sentences in  $L$ , with  $Q(e)$  as a constituent, which are true iff, say,  $e$  begins with the letter “a”, or  $e$  consists of five words, etc. Then we have:

(NQ) *Suppose  $L$  is an interpreted language that has quotation and whose sentence semantics  $\mu$  respects truth values. Then, either  $\mu$  is one-one or it is not compositional.*

For suppose there are distinct  $e, f \in X$  such that  $\mu(e) = \mu(f)$ . Since they have distinct shapes, some true sentence  $s$  in  $X$  with  $Q(e)$  as a constituent is sensitive to this difference, i.e. it becomes false when  $e$  is replaced by  $f$ . There is a frame  $G \in \mathbb{F}$  such that  $s = G(e)$ . Since  $\mu$  respects truth values,  $\mu(G(e)) \neq \mu(G(f))$ .

<sup>1</sup>It is enough to assume here that we can quote sentences. In general, of course, one wants to quote arbitrary expressions, perhaps even arbitrary sequences of atomic symbols.

So  $\mu$  is not compositional. And so  $\equiv_{\mu}^F$  does not coincide with  $\equiv_{\mu}$  on sentences: we have  $e \equiv_{\mu} f$  but  $e \not\equiv_{\mu}^F f$ . Indeed, as remarked in the preceding section, the Fregean semantics becomes trivial.

This is essentially nothing but the familiar ‘opacity’ of quotation, but formulated in general terms which reveal the very minimal assumptions needed about  $L$ ; for example, it doesn’t rely on identifying meaning with reference. There are statements in the literature which appear to contradict (NQ), but on a closer look, they don’t.<sup>2</sup>

What should we conclude? The strategy of weakening the synonymy  $e \equiv_{\mu} f$  doesn’t seem helpful, since respecting truth values looks like a minimum requirement. The remaining alternative is to simply leave out quotation from the language. That is certainly possible. On the other hand, quotation, in the pure form of having a means of referring to linguistic items, is such a natural mechanism with such a straightforward semantics. And if we admit this mechanism in the language, compositionality is lost.

But maybe not completely lost. Section 10 will sketch a generalization of compositionality that admits quotation, and certain other recalcitrant linguistic constructions as well. But first I need to say something about compositionality and *context*.

## 9 Dependence on extra-linguistic context

Context dependence in natural languages is ubiquitous. The clearest case is *indexicals*. Normally one wants to assign a *meaning* to sentences like

- (1) I am hungry.

But if this meaning is to have anything to do with *truth conditions*, you need to account for the fact that the truth of (1) varies with the context of utterance. There are basically two ways to proceed. Either you let the meaning assignment  $\mu$  take expressions *and* contexts as arguments. Or you *curry*, that is, you introduce, in the words of (Lewis, 1980), ‘constant and complex semantic values’, values which themselves are functions from contexts to ordinary meanings.<sup>3</sup> On the curried approach the notion of compositionality as we have defined it applies. But on the first approach we have this extra argument, requiring a

<sup>2</sup>For example, (Potts, 2007) presents an elegant semantics for (not only pure) quotation, which he claims to be compositional. What he in effect does is to give a recursive truth definition whose clauses for complex expressions are not of the form

$$\nu(F(e_1, \dots, e_n)) = h_F(\nu(e_1), \dots, \nu(e_n))$$

but rather

$$\nu(F(e_1, \dots, e_n)) = h_F(\nu(e_1), \dots, \nu(e_n), e_1, \dots, e_n)$$

Thus, the expressions themselves, as well as their meanings, are arguments of the semantic operations. This is much weaker than (homomorphism) compositionality; see also (Pagin & Westerståhl, 2010a), sect. 3.2.

<sup>3</sup>This is the *functional* version. On the *structured* version, meanings are structured objects, possibly with ‘holes’ that can be filled with e.g. contexts. Everything I say below holds, with slight alterations, for the structured approach as well.

slight reformulation. How slight, and what are the relations between the two approaches? Abstractly, the situation is easy to describe.<sup>4</sup>

As before, the language  $L$  has a constituent structure  $(\mathbb{E}, \mathbb{F})$  and a semantics  $\mu$ , but now  $\mu$  is a function from  $\mathbb{E} \times C$  to some set  $Z$  of ‘ordinary’ meanings, where  $C$  is a set of *contexts*. For simplicity, I’ll assume  $\mu$  is total. Contexts can be any objects; typical cases are

- $\mu(\forall x\varphi, f) = \text{T}$  iff for all  $a \in M$ ,  $\mu(\varphi, f(a/x)) = \text{T}$  (contexts as assignments)
- $\mu(P\varphi, t) = \text{T}$  iff for some  $t' < t$ ,  $\mu(\varphi, t') = \text{T}$  (contexts as times)
- $\mu(I, c) = \textit{speaker}_c$  (contexts as utterance situations)

Currying, we get the 1-ary function

$$\mu_{\text{curr}}: \mathbb{E} \longrightarrow [C \longrightarrow Z]$$

( $[X \longrightarrow Y]$  is the set of functions from  $X$  to  $Y$ ), defined by

$$\mu_{\text{curr}}(e)(c) = \mu(e, c)$$

We know what compositionality of  $\mu_{\text{curr}}$  amounts to. For  $\mu$ , there are two slightly different natural notions (using the functional formulation):

### Context-sensitive compositionality

- (i)  $\mu$  is *compositional* iff for each  $F \in \mathbb{F}$  there is an operation  $s_F$  such that for all  $c \in C$ ,

$$\mu(F(e_1, \dots, e_n), c) = s_F(\mu(e_1, c), \dots, \mu(e_n, c))$$

- (ii)  $\mu$  is *weakly compositional* iff for each  $F \in \mathbb{F}$  there is an operation  $s_F$  such that for all  $c \in C$ ,

$$\mu(F(e_1, \dots, e_n), c) = s_F(\mu(e_1, c), \dots, \mu(e_n, c), c)$$

So the only difference is that the context itself is allowed to be an argument of the semantic operations in the weak case. This is actually an important weakening, and allows as compositional several phenomena that are often considered pragmatic rather than semantic. Here is how these notions are related.

### Proposition 1

(Contextual) compositionality of  $\mu$  implies weak (contextual) compositionality of  $\mu$ , which in turn implies (ordinary) compositionality of  $\mu_{\text{curr}}$ , but none of these implications can in general be reversed.

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<sup>4</sup>For a details, motivation, proofs, and discussion of the issues raised in this section, see (Pagin, 2005) and (Westerstahl, 2012). Note that the ‘meanings’ in  $Z$  can themselves be functions, say, from possible worlds to truth values.

The first two examples above, with contexts as assignments and as times, respectively, are typical instances of semantics which are *not* (not even weakly) contextually compositional, but where the curried version *is* compositional. The first of these reflects the familiar fact that Tarski's truth definition for first-order logic is compositional if you take *sets of assignments* (not truth values) as semantic values. The third example, on the other hand, with contexts as utterance situations, you typically expect to belong to a (contextually) compositional semantics. The reason is that in the first two cases contexts are *shifted* in the right-hand side of the clause, but this is usually not thought to happen in the third case.

There is much to say about which notion applies to which kind of linguistic construction, but here the points to take home are these: (a) Compositionality makes perfect sense also when meaning is context-dependent (which is the rule rather than the exception in natural languages). (b) But there are (at least) three distinct notions involved, related as in Proposition 1, and in applications one needs to be aware of which one is at stake.

## 10 General compositionality

Once extra-linguistic context dependence is seen to be compatible with compositionality, there is no reason why linguistic context dependence shouldn't also be. Such dependence can be understood in different ways. One is dependence on other parts of *discourse*, as when an anaphoric pronoun refers back to something introduced earlier by a name or, as in (2), an indefinite description:

- (2) A woman entered the room. Only Fred noticed her.

Here I am interested in dependence on *sentential* context, of the kind Frege talks about in the following well-known passage:

If words are used in the ordinary way, what one intends to speak of is their reference. It can also happen, however, that one wishes to talk about the words themselves or their sense. This happens, for instance, when the words of another are quoted. One's own words then first designate words of the other speaker, and only the latter have their usual reference. We then have signs of signs. In writing, the words are in this case enclosed in quotation marks. Accordingly, a word standing between quotation marks must not be taken to have its ordinary reference. (Frege, 1892):58–9

What Frege says here is that *the type of linguistic context can change the meaning*. Quotation is one such type, sometimes indicated by quotation marks, and in this context, words no longer refer to what they usually refer to, but to themselves. Attitude contexts is of another type (only hinted at in this passage but developed in other parts of (Frege, 1892)); then we use the same words to “talk about . . . their sense.”

In the syntactic algebra framework (section 3.1), terms are construction trees, so you can identify the (linguistic) context of a term occurrence  $t$  in a sentence  $s$  (or any complex term with  $t$  as a subterm) with the unique *path* from the top node to  $t$ . Let a *context typing* be a partition of the set of such paths, with the property that the type of each daughter  $t_i$  of a node  $\alpha(t_1, \dots, t_n)$  is determined by the type of that node,  $\alpha$ , and  $i$ . Then we can formulate compositionality with  $C$  as the set of context types just as we did weak compositionality for arbitrary  $C$ , but with the difference that the meaning of  $\alpha(t_1, \dots, t_n)$  at  $c$  is determined by  $\alpha$ ,  $c$ , and the meanings of the  $t_i$  at  $c_i$ , where  $c_i$  is the context type determined by  $c$ ,  $\alpha$ , and  $i$ .

This version doesn't easily extend to the constituent structure framework (section 3.2), but there is another formulation, equivalent to the one just sketched for syntactic algebras, but applying more generally.<sup>5</sup> In the constituent structure framework, the idea would be to let a semantics be a *set*  $S$  of mappings from  $\mathbb{E}$  to meanings, together with a *selection function*  $\Psi$ , telling which function  $\mu_i \in S$  should be applied to  $e_i$  when  $\mu$  applies to  $F(e_1, \dots, e_n)$ . Thus, *compositionality of*  $(S, \Psi)$  is the property that for each  $F \in \mathbb{F}$  and each  $\mu \in S$  there is an operation  $r_{\mu, F}$  such that when  $F(e_1, \dots, e_n) \in \mathbb{E}$ ,

$$\mu(F(e_1, \dots, e_n)) = r_{\mu, F}(\mu_1(e_1), \dots, \mu_n(e_n)),$$

where  $\mu_i = \Psi(\mu, F, i)$ . So there is no extra argument to the meaning assignment, but instead there may be more than one meaning assignment function. We call this *general compositionality*. (If  $S$  is a unit set we have the ordinary notion.)

The application to quotation is now straightforward: in the simplest version you just need two meaning assignment functions, a *default* function  $\mu_d$  and a *quotation* function  $\mu_q$ , and the quote frame  $Q$  (section 8) has the property that whatever function is applied to  $Q(e)$ ,  $\mu_q$  is applied to  $e$ . And of course, for all  $e \in \mathbb{E}$ ,  $\mu_q(e)$  is (the surface representation of)  $e$  itself.

The idea of a semantics that allows switching between different meaning assignments appears quite natural, not only for quotation but for certain other linguistic phenomena as well.<sup>6</sup> Frege had a similar idea for attitude contexts. (Glüer & Pagin, 2006, 2008, 2012) use such a semantics for the modal operators, to deal with rigidity phenomena without treating names or natural kind terms as rigid designators. The point here has just been to show that compositionality, in its general form, is still a viable issue for such semantics.

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[During the course I have, in addition to the references below, mentioned (at least) the following ones:

<sup>5</sup>This formulation is due to Peter Pagin. For full details of these notions of compositionality (in the syntactic algebra setting), their properties, and the application to quotation, see (Pagin & Westerståhl, 2010b).

<sup>6</sup>According to (Hodges, 2012):249, this is the notion of *tahrīf* used by the eleventh-century Persian-Arabic writer Ibn Sīnā ('Avicenna'), among others. Hodges is skeptical of its usefulness in semantics (pp. 255–6), but at least the application to quotation (not discussed by him) seems very natural.

(Barker & Jacobson, 2007), (Barwise, 1979), (Dowty, 2007), (Cameron & Hodges, 2001), (Fodor, 2001), (Frege, 1884, 1923), (Heim & Kratzer, 1998), (Higginbotham, 1986, 2007), (Hintikka & Sandu, 1997), (Hodges, 1997, 2001, 2005, 2011), (Horwich, 2005), (Jacobson, 2002), (Kracht, 2007), (Montague, 1974 (1970), 1974), (Pelletier, 1994), (Väänänen, 2007), (Westerståhl, 2002), (Zadrozny, 1994)

So this reference should be pretty complete for the course.]

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