Compositionality I: definitions and variants*

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Abstract

This is the first part of a two-part article on semantic compositionality, i.e. the principle that the meaning of a complex expression is determined by the meanings of its parts and the way they are put together. Here we provide a brief historical background, a formal framework for syntax and semantics, precise definitions, and a survey of variants of compositionality. Stronger and weaker forms are distinguished, as well as generalized forms that cover extra-linguistic context dependence as well as linguistic context dependence. In the second article we survey arguments for and arguments against the claim that natural languages are compositional, and consider some problem cases. It will be referred to as Part II.

1 Background

Compositionality is a property that a language may have and may lack, namely the property that the meaning of any complex expression is *determined* by the meanings of its parts and the way they are put together. The language can be natural or formal, but it has to be interpreted. That is, meanings, or more generally, *semantic values* of some sort must be assigned to linguistic expressions, and compositionality concerns precisely the distribution of these values.

Particular semantic analyses that are in fact compositional were given already in antiquity,[1] but apparently without any corresponding general conception. Notions that approximate the modern concept of compositionality did

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[1]For instance, in *Sophist*, chapters 24-26, Plato discusses subject-predicate sentences, and suggests (pretty much) that such a sentence is true [false] if the predicate (verb) attributes to what the subject (noun) signifies things that are [are not].

emerge in medieval times. In the Indian tradition, in the 4th or 5th century CE, Śabara says that

> The meaning of a sentence is based on the meaning of the words.

and this is proposed as the right interpretation of a sūtra by Jaimini from sometime 3rd-6th century BCE (cf. Houben 1997, 75-76). The first to propose a general principle of this nature in the Western tradition seems to have been Peter Abelard (Abelard 2008, 3.00.8) in the first half of the 12th century, saying that

> Just as a sentence materially consists in a noun and a verb, so too the understanding of it is put together from the understandings of its parts.[2]

Abelard's principle directly concerns only subject-predicate sentences, it concerns the understanding process rather than meaning itself, and he is unspecific about the nature of the putting-together operation. The high scholastic conception is different in all three respects. In early middle 14th century John Buridan (Buridan 1998, 2.3, Soph. 2 Thesis 5, QM 5.14, fol. 23vb) states what has become known as the *additive principle*:

> The signification of a complex expression is the sum of the signification of its non-logical terms.[3]

The additive principle, with or without the restriction to non-logical terms, appears to have become standard during the late middle ages.[4] The medieval theorists apparently did not possess the general concept of a function, and instead proposed a particular function, that of summing (collecting). Mere collecting is inadequate, however, since the sentences *All A's are B's* and *All B's are A's* have the same parts, hence the same collection of part-meanings and hence by the additive principle have the same meaning.

With the development of mathematics and concern with its foundations came a renewed interest in semantics. Gottlob Frege is generally taken to be the first person to have formulated explicitly the notion of compositionality and to claim that it is an essential feature of human language.[5] In "Über Sinn und Bedeutung", 1892, he writes:

[2] Translation by and information from Peter King (2007, 8).

[3] Translation by and information from Peter King (2001, 4).

[4] In 1500, Peter of Ailly refers to the common view that it 'belongs to the [very] notion of an expression that every expression has parts each one of which, when separated, signifies something of what is signified by the whole.' (Ailly 1980, 30).

[5] Some writers have doubted that Frege really expressed, or really believed in, compositionality; e.g. Pelletier 2001 and Janssen 2001.

> Let us assume for the time being that the sentence has a reference. If we now replace one word of the sentence by another having the same reference, this can have no bearing upon the reference of the sentence. (Frege 1892, p. 62)

This is (a special case of) the *substitution version* of the idea of semantic values being determined; if you replace parts by others with the same value, the value of the whole doesn't change. Note that the values here are *Bedeutungen* (referents), such as truth values (for sentences) and individual objects (for individual-denoting terms).

Both the substitution version and the function version (see below) were explicitly stated by Rudolf Carnap in Carnap 1956, p. 121 (for both extension and intension), and labeled 'Frege's Principles of Interchangeability'. The term 'compositional', used in a similar sense, to characterize meaning and understanding, derives from Jerry Fodor and Jerrold Katz (1964), with reference to Chomsky but not to Frege or Carnap.

Today, compositionality is a key notion in linguistics, philosophy of language, logic, and computer science, but there are divergent views about its exact formulation, methodological status, and empirical significance. To begin to clarify some of these views we need a framework for talking about compositionality that is sufficiently general to be independent of particular theories of syntax or semantics and yet allows us to capture the core idea behind compositionality.

2 A framework

The function version and the substitution version of compositionality are two sides of the same coin: that the meaning (value) of a compound expression is a *function* of certain other things (other meanings (values) and a 'mode of composition'). As we will see presently, the substitution version is slightly more general and versatile. To formulate these versions, two things are needed: a set of *structured expressions* and a *semantics* for them.

Structure is readily taken as *algebraic* structure, so that the set E of linguistic expressions is a domain over which certain operations (syntactic rules) are defined, and moreover E is *generated* by these operations from a subset A of *atoms* (primitive expressions, e.g. words). In the literature there are essentially two ways of fleshing out this idea. One, which originates with Montague,[6] takes as primitive the fact that linguistic expressions are grouped into *categories* or

[6]See Montague 1974*a*, in particular the paper 'Universal grammar' from 1970.

sorts, so that a syntactic rule comes with a specification of the sorts of each argument as well as of the value. This use of a *many-sorted algebra* as an abstract linguistic framework is described in Janssen 1986 and Hendriks 2001. The other approach, first made precise in Hodges 2001, is one-sorted but uses *partial algebras* instead, so that rather than requiring the arguments of an operation to be of certain sorts, the operation is simply *undefined* for unwanted arguments. The partial approach is in a sense more general than the many-sorted one, as well as easier to formulate, and we follow it here.[7]

Thus, let a *grammar*

$$\mathbf{E} = (E, A, \Sigma)$$

be a partial algebra, where E and A are as above and Σ is a set that, for each required $n \geq 1$, has a subset of partial functions from E^n to E, and is such that E is generated from A via Σ. To illustrate, the familiar rules

$$\text{NP} \longrightarrow \text{Det N} \qquad \text{(NP-rule)}$$
$$\text{S} \longrightarrow \text{NP VP} \qquad \text{(S-rule)}$$

correspond to binary partial functions, say $\alpha, \beta \in \Sigma$, such that, if *most*, *dog*, and *bark* are atoms in A, one derives as usual the sentence *Most dogs bark* in E, by first applying α to *most* and *dog*, and then applying β to the result of that and *bark*. These functions are necessarily partial; for example, β is undefined whenever its second argument is *dog*.[8]

Both in the partial and in the many-sorted framework it may happen that one and the same expression can be generated in more than way, i.e. the grammar may allow *structural ambiguity*. Also, it may happen that a semantically relevant element is not represented in the surface expression.

So in the most general case, it is not really the expressions in E but rather their derivation histories, or 'analysis trees', that should be assigned semantic

[7] A many-sorted algebra can in a straightforward way be turned into a one-sorted partial one (but not always vice versa), and under a natural condition the sorts can be recovered in the partial algebra. See Westerståhl 2004 for further details and discussion. Note also that some theorists combine partiality with primitive sorts; for example, Keenan and Stabler 2004 and Kracht 2007.

[8] Note that for a speaker to have a grasp of an infinite syntax by finite means, rules such as the NP-rule and the S-rule must *hold* in the sense that it is part of a speaker's competence e.g. that for any pair of terms (t, u) for which the operator α is defined, $\alpha(t, u)$ is an appropriate second argument to β.

We can call a grammar $\mathbf{E} = (E, A, \Sigma)$ *inductive* if there is a finite partition $(E_s)_{s \in S}$ of E such that it holds of each $\alpha \in \Sigma$ that its range is a subset of some E_{s_i} and its domain is a cartesian product $E_{s_1} \times \ldots \times E_{s_n}$ of sets in $(E_s)_{s \in S}$. That grammars are inductive in this sense is a natural requirement on syntax, but it is not necessary for the semantics to be compositional.

values. These derivation histories are conveniently represented by the terms in the *term algebra* corresponding to $\mathbf{E}$. The sentence itself, i.e. the value of applying the syntactic functions as above, could be identified with a string of words (sounds, phonemes,...), but its derivation history is represented by the *term*

$$t = \beta(\alpha(\mathit{most}, \mathit{dog}), \mathit{bark})$$

in the term algebra. The term algebra is partial too: the *grammatical terms* are those where all the functions involved are defined for the respective arguments. So t is grammatical but $\beta(\alpha(\mathit{most}, \mathit{dog}), \mathit{dog})$ is not. Let $GT_{\mathbf{E}}$ be the set of grammatical terms for $\mathbf{E}$.[9]

Note that the symbols 'α', 'β',... do a double duty here: they name elements of Σ, i.e., partial functions from expressions to expressions, and these very names are used in the term algebra. For example, we assumed above that

$$\alpha(\mathit{most}, \mathit{dog}) = \mathit{most\ dogs}$$

but this equation only makes sense if α is a function, which applied to two elements of E — in this case, the atoms *most* and *dog* — yields as value another element of E — in this case, the string *most dogs*.[10] However, the *term* $\alpha(\mathit{most}, \mathit{dog})$ doesn't belong to E but to the term algebra. Sometimes one needs to reflect this distinction in the notation; we shall then use symbols with bars over them as names of those symbols. With that notation, we have

$$\alpha(\mathit{most}, \mathit{dog}) \in E \text{ and } \overline{\alpha}(\overline{\mathit{most}}, \overline{\mathit{dog}}) \in GT_{\mathbf{E}}.$$

Each term in $GT_{\mathbf{E}}$ corresponds to a unique string in E. Thus, there is a *string value function* V from $GT_{\mathbf{E}}$ to E. For a simple term like $\overline{\mathit{most}}$, $V(\overline{\mathit{most}}) = \mathit{most}$, the corresponding expression. In case we need to distinguish between

[9] This is relevant for the question of the compositionality of thought. For thought to have a compositional semantics it first needs a system of mental representations with constituent structure. This would seem to require a Language of Thought (LOT), in the sense of Fodor 1987, 2008, where a mental concept F is a constituent of a mental concept G just in case F is always tokened when G is. However, if the constituent structure is an *underlying* structure rather than a surface structure, then it can be constituted by other relations than co-tokening between the mental concepts. For instance, Werning 2005*a* defines constituent structure for so-called oscillatory connectionist networks.

[10] More correctly, we should write the string value of $\alpha(\mathit{most}, \mathit{dog})$ as $\mathit{most}^\frown\llcorner\!\lrcorner^\frown \mathit{dogs}$, where '$\llcorner\!\lrcorner$' denotes word space and '$^\frown$' concatenation, but the simplified notation used here is easier to read. Of course, if we were to theorize about spoken language, this treatment would have to be changed.

homonymous simple terms, like $\overline{bank_1}$ and $\overline{bank_2}$, we will have

$$V(\overline{bank_1}) = V(\overline{bank_2}) = bank.$$

For a complex term $\overline{\alpha}(t_1, \ldots, t_n)$ (using now the above notation) we have

$$V(\overline{\alpha}(t_1, \ldots, t_n)) = \alpha(V(t_1), \ldots, V(t_n)),$$

where α is defined for the arguments $V(t_1), \ldots, V(t_n)$ precisely when the term $\overline{\alpha}(t_1, \ldots, t_n)$ is grammatical.[11] To illustrate:

$$\begin{aligned}
V(\overline{\beta}(\overline{\alpha}(\overline{most}, \overline{dog}), \overline{bark})) &= \beta(V(\overline{\alpha}(\overline{most}, \overline{dog})), V(\overline{bark}) \\
&= \beta(\alpha(V(\overline{most}), V(\overline{dog})), bark) \\
&= \beta(\alpha(most, dog), bark) \\
&= \beta(most\ dogs\ , bark) \\
&= most\ dogs\ bark
\end{aligned}$$

The second thing needed to talk about compositionality is a *semantics* for **E**. The semantics is naturally taken to be a function μ to some set M of semantic values ('meanings'). It is most simple and straightforward to let (a subset of) $GT_{\mathbf{E}}$ be the domain of μ. That is, μ maps grammatical terms on meanings.[12] That terms are the arguments to μ does not mean that the expressions themselves are meaningless, only that an expression has meaning derivatively, relative to a way of constructing it, i.e. to a corresponding grammatical term. Indeed, one often slurs over the difference, writing $\mu(e)$ for an expression e in E, when what one really should have written is $\mu(t)$ for some grammatical term t with $V(t) = e$.

There are several reasons why the semantic function μ should be allowed to be partial, too. For example, it may represent our partial understanding of some language, or our attempts at a semantics for a fragment of a language. Further, even a complete semantics will be partial if one wants to maintain a distinction between *meaningfulness* (being in the domain of μ) and *grammaticality* (being

[11] In other words, V is a homomorphism from the term algebra to the expression algebra **E**.

[12] There are alternatives. One is to take *disambiguated* expressions from E: expressions somehow annotated to resolve syntactic ambiguities. Phrase structure markings by means of labeled brackets are of this kind. Another option is to have an extra syntactic level, like LF in the Chomsky school, as the semantic function domain. The choice between such alternatives is largely irrelevant from the point of view of compositionality, as long as the syntactic arguments have the required constituent structure. Note, however, that there is no string value function from LF to surface form, since two distinct strings may be considered to have the same LF; examples could be *John loves Susan* and *Susan is loved by John*.

derivable by the grammar rules).

No assumption is made about meanings. In the abstract framework, the nature of the meanings does not matter more than what is required to determine the relation of *synonymy*: define, for $u, t \in E$,

$$u \equiv_\mu t \text{ iff } \mu(u), \mu(t) \text{ are both defined and } \mu(u) = \mu(t).$$

$\equiv_\mu$ is a partial equivalence relation on E. All that is relevant for compositionality itself is captured by properties of this equivalence relation.

This point deserves to be emphasized. It is often claimed that the concept of compositionality remains underspecified as long as we are not told what the syntax is like and what the semantic values are, but this is not correct. The notion of compositionality itself is independent of how these are chosen: it is a formal property of a semantics relative to a syntax. It may well happen that the assignment of one kind of semantic values to expressions is compositional while the assignment of other values is non-compositional.[13] Likewise, a change of syntactic analysis may restore compositionality. Even if compositionality in itself is regarded as a desirable property (see Part II for discussion), the evaluation of a proposed account must also factor in the reasonableness of the syntactic analysis and the semantic values chosen. As we will see in the next section, it is always possible to enforce compositionality by *un*reasonable means, but this fact is irrelevant to the question of whether there exist or not reasonable accounts of certain linguistic phenomena that satisfy the principle of compositionality.

3 Variants and properties

3.1 Basic compositionality

We can now easily formulate both the function version and the substitution version of compositionality, given a grammar **E** and a semantics μ as above.

Funct(μ) For every rule $\alpha \in \Sigma$ there is a meaning operation r_α such that if $\alpha(u_1, \ldots, u_n)$ has meaning, $\mu(\alpha(u_1, \ldots, u_n)) = r_\alpha(\mu(u_1), \ldots, \mu(u_n))$.

[13]For a familiar example, Frege noted that if a sentence's *Bedeutung* is its truth value, attitude reports are not compositional. His suggestion to use other semantic values for thus embedded sentences (namely, their *Sinn*) can be seen as a way to restore compositionality. Another familiar example concerns predicate logic: the standard truth definition for the language of predicate logic gives a semantics which is not compositional with respect to truth value as semantic value, but which is compositional with respect to sets of variable assignments (or functions from assignments to truth values). Cf. Janssen 1997, Westerståhl 2009.

A variant is to use, for each $n \geq 1$, an $(n+1)$-ary operation r_n instead and let α itself be its first argument, or again a single functional r such that for each α, $r(\alpha) = r_\alpha$. Note that Funct(μ) presupposes the *Domain Principle* (DP): subterms of meaningful terms are also meaningful.

The substitution version of compositionality is given by

Subst($\equiv_\mu$) If $s[u_1, \ldots, u_n]$ and $s[t_1, \ldots, t_n]$ are both meaningful terms, and if $u_i \equiv_\mu t_i$ for $1 \leq i \leq n$, then $s[u_1, \ldots, u_n] \equiv_\mu s[t_1, \ldots, t_n]$.

Here the notation $s[u_1, \ldots, u_n]$ indicates that the term s contains—not necessarily immediate—*disjoint* occurrences of subterms among $u_1, \ldots, u_n$, and $s[t_1, \ldots, t_n]$ results from replacing each u_i by t_i.[14] Subst($\equiv_\mu$) does not presuppose DP. In this respect it is more general, for one can easily think of semantics for which DP fails. However, a first observation is:

(1) Under DP, Funct(μ) and Subst($\equiv_\mu$) are equivalent.[15]

The requirements of basic compositionality are in some respects not so strong, as can be seen from the following observations:

(2) If μ gives the same meaning to every expression, then Funct(μ) holds.
(3) If μ gives different meanings to all expressions, then Funct(μ) holds.

(2) is of course trivial. For (3), consider Subst($\equiv_\mu$) and observe that if no two expressions have the same meaning, then $u_i \equiv_\mu t_i$ entails $u_i = t_i$, so Subst($\equiv_\mu$), and therefore Funct(μ), hold trivially.

3.2 Recursive semantics

The function version of compositional semantics is given by recursion over syntax, but that does not imply that the meaning operations are defined by re-

[14]Restricted to immediate subterms, Subst($\equiv_\mu$) says that $\equiv_\mu$ is a (partial) *congruence relation*:

> If $\alpha(u_1, \ldots, u_n)$ and $\alpha(t_1, \ldots, t_n)$ are both meaningful and $u_i \equiv_\mu t_i$ for $1 \leq i \leq n$, then $\alpha(u_1, \ldots, u_n) \equiv_\mu \alpha(t_1, \ldots, t_n)$.

Under DP, this is equivalent to the unrestricted version.

[15]That Funct(μ) implies Subst($\equiv_\mu$) is obvious when Subst($\equiv_\mu$) is restricted to immediate subterms, and otherwise proved by induction over the complexity of terms. In the other direction, the operations r_α must be found. For $m_1, \ldots, m_n \in M$, let $r_\alpha(m_1, \ldots, m_n) = \mu(\alpha(u_1, \ldots, u_n))$ if there are terms u_i such that $\mu(u_i) = m_i$, $1 \leq i \leq n$, and $\mu(\alpha(u_1, \ldots, u_n))$ is defined. Otherwise, $r_\alpha(m, \ldots, m_n)$ can be undefined (or arbitrary). This is enough, as long as we can be certain that the definition is independent of the choice of the u_i, but that is precisely what Subst($\equiv_\mu$) says.

cursion over *meaning*, in which case we have *recursive semantics*. Standard semantic theories are typically both recursive and compositional, but the two notions are mutually independent. For a semantic function μ to be given by recursion it must hold that:

Rec(μ) There is a function b and for every $\alpha \in \Sigma$ an operation r_α such that for every meaningful expression s,

$$\mu(s) = \begin{cases} b(s) & \text{if } s \text{ is atomic} \\ r_\alpha(\mu(u_1), \ldots, \mu(u_n), u_1, \ldots, u_n) & \text{if } s = \alpha(u_1, \ldots, u_n) \end{cases}$$

For μ to be recursive, the basic function b and the meaning composition operation r_α must themselves be recursive, but this is not required in the function version of compositionality. In the other direction, the presence of the terms $u_1, \ldots, u_n$ themselves as arguments to r_α, has the effect that the compositional substitution laws need not hold.[16]

Note that if we drop the recursiveness requirement on b and r_α, Rec(μ) becomes vacuous. This is because $r_\alpha(m_1, \ldots, m_n, u_1, \ldots, u_n)$ can simply be *defined* to be $\mu(\alpha(u_1, \ldots, u_n))$ whenever $m_i = \mu(u_i)$ for all i and $\alpha(u_1, \ldots, u_n)$ is meaningful (undefined otherwise). Since inter-substitution of synonymous terms changes at least one argument of r_α, no counterexample is possible.

3.3 Weaker versions

Basic (first-level) compositionality takes the meaning of a complex term to be determined by the meanings of the *immediate* sub-terms and the top-level syntactic operation. We get a weaker version—*second-level* compositionality—if we require only that the operations of the *two* highest levels, together with the meanings of terms at the second level, determine the meaning of the whole complex term.[17] *Third-level* compositionality is defined analogously, and is weaker

[16] This can happen e.g. with simple semantics for quotation, as noted e.g. in Werning 2005*b*. Such a semantics is given by Christopher Potts (2007), incorrectly claiming that it is compositional. Similarly, it is pointed out by Zoltán Szabó (2007, note 25) that compositionality is violated in some accounts of belief sentences that appeal to interpreted logical forms. For further instructive remarks, see also Janssen 1997.

[17] A possible example, from Peters and Westerståhl 2006, ch. 7, concerns possessive determiner phrases like *some student's*, taken to be generated by (NP-rule) above and

$$\text{Det} \longrightarrow \text{NP 's} \qquad \text{(Poss)}$$

If the semantic value of the Det in (NP-rule) is a type $\langle 1, 1\rangle$ quantifier Q and the value of N is a set C, the value of the resulting NP is arguably the type $\langle 1\rangle$ quantifier Q^C, i.e. Q with its restriction argument *frozen* to C. Peters and Westerståhl argue that when this still. In the extreme case we have *bottom-level*, or *weak* functional compositionality, if the meaning the complex term is determined only by the meanings of its atomic constituents and the total syntactic construction (i.e. the derived operation that is extracted from a complex term by knocking out the atomic constituents). A function version of this is somewhat cumbersome to formulate precisely (but see Hodges 2001, sect. 5),[18] whereas the substitution version becomes simply:

AtSubst($\equiv_\mu$) Just like Subst($\equiv_\mu$) except that the u_i and t_i are all atomic.

Although weak compositionality is not completely trivial (a language could lack the property), it does not serve the language users very well: the meaning operation r_α that corresponds to a complex syntactic operation α cannot be predicted from its build-up out of simpler syntactic operations and their corresponding meaning operations. Hence, there will be infinitely many complex syntactic operations whose semantic significance must be learned one by one.

3.4 Stronger versions

We get stronger versions of compositionality by enlarging the domain of the semantic function, or by placing additional restrictions on meaningfulness or on meaning composition operations. An example of the first is Zoltan Szabo's idea (Szabó 2000) that the same meaning operations define semantic functions in all possible human languages, not just for all sentences in each language taken by itself. That is, whenever two languages have the same syntactic operation, they also associate the same meaning operation with it.

An example of the second option is what Wilfrid Hodges has called the *Husserl property* (going back to ideas in Husserl 1900):

(Huss) Synonymous terms belong to the same (semantic) category.

Here the notion of category is defined in terms of substitution; say that $u \sim_\mu t$

complex NP is the argument in the (Poss) rule, a determination of the semantic value of the resulting possessive determiner requires access to both Q and C (due to the phenomenon of so-called *narrowing*); i.e. it would require a unique *decomposition* of Q^C, but this is in general not possible (see Westerståhl 2008 for decomposition of quantifiers, also in connection with possessives). If so, the corresponding semantics is at most second-level compositional.

[18] Terminology concerning compositionality is somewhat fluctuating. David Dowty (2007) calls (an approximate version of) weak functional compositionality *Frege's Principle*, and refers to Funct(μ) as *homomorphism compositionality*, or *strictly local compositionality*, or *context-free semantics*. In Larson and Segal 1995, this is called *strong compositionality*. The labels *second-level compositionality*, *third-level*, etc. are not standard in the literature but seem appropriate.

if, for every term s in E, $s[u] \in dom(\mu)$ iff $s[t] \in dom(\mu)$. So (Huss) says that synonymous terms can be inter-substituted without loss of meaningfulness. This is often a reasonable requirement. (Huss) also has the consequence that $\mathrm{Subst}(\equiv_\mu)$ can be simplified to $\mathrm{Subst}_1(\equiv_\mu)$, which only deals with replacing *one* subterm by another. Then one can replace n subterms by applying $\mathrm{Subst}_1(\equiv_\mu)$ n times; (Huss) guarantees that all the 'intermediate' terms are meaningful.

An example of the third kind is that of requiring the meaning composition operations to be recursive, or computable. To make this idea more precise, in analogy with arithmetic, we need to impose more order on the meaning domain. We have to view meanings as themselves given by an algebra $\mathbf{M} = (M, B, \Omega)$, where $B \subseteq M$ is a finite set of *basic meanings*, Ω is a finite set of elementary operations from n-tuples of meanings to meanings, and M is generated from B by means of the operations in Ω. This allows the definition of functions by recursion over M, and the meaning operations are to be of this kind (those in Ω will correspond to the successor operation for ordinary recursion over natural numbers). The semantic function μ is then defined simultaneously by recursion over syntax and by recursion over the meaning domain. Assuming that the elementary meaning operations are computable in a sense relevant to cognition, the semantic function itself is computable.

A further step in this direction is to require that the meaning operation relevant to semantics are of some restricted kind that makes them easy to compute, and thereby reduces or minimizes the (time) complexity of semantic interpretation. Requiring that meaning operations are *polynomial*, i.e. either elementary or formed from elementary operations by function composition, is the most natural restriction of this kind.[19]

Another strengthening, also introduced in Hodges 2001, concerns Frege's so-called Context Principle. A famous but cryptic saying by Frege in Frege 1884 is: "Never ask for the meaning of a word in isolation, but only in the context of a sentence" (p. x). This principle has been much discussed in the literature[20], and often taken to *conflict* with compositionality. However, if not seen as saying that words somehow lose their meaning in isolation, it can be interpreted as a *constraint* on meanings, in the form of what we might call the Contribution Principle, roughly:

(CP) The meaning of a term is the contribution it makes to the meanings of complex terms of which it is a part.

[19] Cf. Pagin 2009 for work in this direction.

[20] For example, Dummett 1973, Dummett 1981, Janssen 2001, and Pelletier 2001.

This is still vague, but Hodges notes that it can be made precise in the form of an additional requirement on the synonymy $\equiv_\mu$. Assuming (Huss), as Hodges does here, consider:

InvSubst$_\exists(\equiv_\mu)$ If $u \not\equiv_\mu t$ then there is some term s such that either exactly one of $s[u]$ and $s[t]$ are meaningful, or both are and $s[u] \not\equiv_\mu s[t]$.

This entails that if two terms of the same category are such that no complex term of which the first is a part changes meaning when the first is replaced by the second, they are synonymous. That is, if they make the *same* contribution to *all* such complex terms, their meanings cannot be distinguished. This can be taken as one half of (CP), and compositionality in the form of Subst$_1(\equiv_\mu)$ as the other.[21]

We can take a step further in this direction by requiring that substitution of terms by terms with *different* meanings *always* changes meaning:

InvSubst$_\forall(\equiv_\mu)$ If for some i, $0 \leq i \leq n$, $u_i \not\equiv_\mu t_i$, then for every term $s[u_1, \ldots, u_n]$ it holds that either exactly one of $s[u_1, \ldots, u_n]$ and $s[t_1, \ldots, t_n]$ are meaningful, or both are and $s[u_1, \ldots, u_n] \not\equiv_\mu s[t_1, \ldots, t_n]$.

This principle disallows synonymy between complex terms that can be transformed into each other by substitution of constituents at least some of which are non-synonymous, but it allows two terms with different structure to be synonymous. Carnap's principle of synonymy as *intensional isomorphism* forbids this, too. With the concept of *intension* from possible-worlds semantics it can be stated as

(RC) $t \equiv_\mu u$ iff
(i) t, u are atomic and co-intensional, or
(ii) for some α, $t = \alpha(t_1, \ldots, t_n)$, $u = \alpha(u_1, \ldots, u_n)$, and $t_i \equiv_\mu u_i$, $1 \leq i \leq n$

[21] Hodges' main application of these notions (Hodges 2001) is to what has become known as the *extension problem*: given a partial compositional semantics μ, under what circumstances can μ be extended to a larger fragment of the language? Here (CP) can be used as a requirement, so that the meaning of a new word w, say, must respect the (old) meanings of complex terms of which w is a part. This is especially adapted to situations when all new items are parts of terms that already have meanings (cofinality). Hodges defines a corresponding notion of *fregean extension* of μ, and shows that in the situation just mentioned, and given that μ satisfies (Huss), a *unique* fregean extension always exists.

Another version of the extension problem is solved in Westerståhl 2004. An abstract account of compositional extension issues is given in Fernando 2005.

(RC) entails both Subst($\equiv_\mu$) and InvSubst$_\forall$($\equiv_\mu$), but is very restrictive. It disallows synonymy between *brother* and *male sibling* as well as between *John loves Susan* and *Susan is loved by John*, and allows different terms to be synonymous only if they differ at most in being transformed from each other by substitution of synonymous atomic terms.[22]

This seems too strong. We get an intermediate requirement as follows. First we define two terms t and u to be *μ-congruent*, $t \simeq_\mu u$:

($\simeq_\mu$) $t \simeq_\mu u$ iff
(i) t or u is atomic, $t \equiv_\mu u$, and neither is a constituent of the other, or
(ii) $t = \alpha(t_1, \ldots, t_n)$, $u = \beta(u_1, \ldots, u_n)$, $t_i \simeq u_i$, $1 \leq i \leq n$, and for all $s_1, \ldots, s_n$, $\alpha(s_1, \ldots, s_n) \equiv_\mu \beta(s_1, \ldots, s_n)$, if either is defined.

Then we require synonymous term to be congruent:

(Cong) If $t \equiv_\mu u$, then $t \simeq_\mu u$.

By (Cong), synonymous terms cannot differ much syntactically, but they may differ in the two crucial respects forbidden by (RC). That (Cong) holds for natural language is a hypothesis. It clearly does not if distinct but logically equivalent sentences are synonymous, but this is usually not accepted.

It is a consequence of (Cong) that meanings are structured entities or can be represented as structured entities, i.e. entities uniquely determined by how they are built, i.e. again entities from which constituents can be extracted. That is, we have projection operations:

(Rev) For every meaning operation $r : E^n \longrightarrow E$ there are projection operations $s_{r,i}$ such that $s_{r,i}(r(m_1, \ldots, m_n)) = m_i$.

(Rev) alone tells us nothing about the semantics. Only together with the fact that the operations r_i are meaning operations for a compositional semantic function μ do we get semantic consequences. The main consequence is that we

[22] More precisely, this holds in any framework, like the present, where the *same* grammatical terms are mapped both on surface strings and on semantic values. In other syntactic frameworks, like that of the Chomsky school, where a distinct level of syntactic representation (such as LF) is directly relevant for semantics, two terms that differ more than allowed by (RC) may still correspond to the same term at the semantically relevant syntactic level (cf. note 12). In such frameworks, unlike Carnap's own, the two pairs mentioned are allowed to be synonymy pairs. By contrast, the suggestion below (Cong) is to allow terms that differ more than is allowed by (RC) to be mapped *directly* on the same semantic value. The end result is the same.

also have a kind of *inverse* functional compositionality:

InvFunct(μ) The syntactic expression of a complex meaning m is determined, up to μ-congruence, by the composition of m and the syntactic expressions of its parts.

For the philosophical significance of inverse compositionality, see sections 1.6 and 2.2 of Part II.[23]

3.5 Direct and indirect compositionality

The terms or derivation trees that are the arguments of the semantic function may differ more or less from the expressions (strings of symbols) that correspond to them. In Jacobson 2002, Pauline Jacobson distinguishes between *direct* and *indirect* compositionality, according to the relation between terms and expressions, as well as between *strong direct* and *weak direct* compositionality. Informally, in strong direct compositionality, expressions are built up from sub-expressions simply by means of concatenation, left or right. In weak direct compositionality, one expression may wrap around another (as *call up* wraps around *him* in *call him up*). As we understand Jacobson, the following defines her strong direct compositionality. Let $V(t)$ (as before) be the expression (string) that corresponds to the grammatical term t, and likewise the occurrence of a string that corresponds to the occurrence of a term in a larger term. Distinct occurrences of terms correspond to distinct occurrences of strings. Then we can state:

(SDC) A language is *strongly directly compositional* iff

- i) For any subterm occurrence t' of a complex grammatical term t, $V(t')$ is a substring occurrence of $V(t)$, and
- ii) for every symbol occurrence x in $V(t)$ there is a proper subterm t'' of t such that x is in $V(t'')$.
- iii) There is a (total) compositional semantic function μ defined on the grammatical terms.[24]

[23] For ($\simeq_\mu$), (Cong), InvFunct(μ), and a proof that (Rev) is a consequence of (Cong) (really of the equivalent statement that the meaning algebra is a *free algebra*), see Pagin 2003. (Rev) seems to be what Jerry Fodor understands by 'reverse compositionality' in e.g. Fodor 2000, p. 371.

[24] Note that for Jacobson, as for Kracht (see next subsection), the arguments of the semantic function are really grammatical terms formed from expression *triples* $\langle$phonology, category, meaning$\rangle$, but this does not essentially change the situation.

The weak direct version is like the strong version except that substrings are allowed to have discontinuous occurrences: every symbol occurrence in the contained string has an occurrence in the containing string and the order between symbol occurrences is preserved, but symbol occurrences from other string occurrences may intervene. For *indirect* compositionality, i.e. for our notion of compositionality here, both conditions i) and ii) (as well as the totality requirement on μ) are dropped: syntactic operations may delete strings, reorder strings, make substitutions and add new elements. In addition, Jacobson distinguishes as more radically indirect theories in which the arguments to the semantic function belong to an indirectly derived syntactic level, like LF in the Chomsky tradition.

Strictly speaking, the direct/indirect distinction is not a distinction between kinds of semantics, but between kinds of syntax. Still, discussion of it tends to focus on the role of compositionality in linguistics, e.g. whether to let the choice of syntactic theory be guided by compositionality (cf. Dowty 2007 and Kracht 2007).[25]

3.6 Expression triples

Some linguists, among them Jacobson, tend to think of grammar rules as applying to *signs*, where a sign is a triple $\langle e, k, m \rangle$ consisting of a string, a syntactic category, and a meaning. This is formalized by Marcus Kracht (see Kracht 2003, Kracht 2007), who defines an *interpreted language* to be a set L of signs in this sense, and a *grammar* G as a set of partial functions (of various arities) from signs to signs, such that L is generated by the functions in G from a subset of atomic (lexical) signs. Thus, a meaning assignment is built into the language, and grammar rules are taken to apply to meanings as well.

This looks like a potential strengthening of our notion of grammar, but is not really used that way, partly because the grammar is taken to operate independently (though in parallel) at each of the three levels. Let p_1, p_2, and p_3 be the projection functions on triples yielding their first, second, and third elements, respectively. Kracht calls a grammar *compositional* if for each n-ary grammar rule α there are three operations $r_{\alpha,1}$, $r_{\alpha,2}$, and $r_{\alpha,3}$ such that for all signs $\sigma_1, \ldots, \sigma_n$ for which α is defined,

$$\alpha(\sigma_1, \ldots, \sigma_n) =$$
$$\langle r_{\alpha,1}(p_1(\sigma_1), \ldots, p_1(\sigma_n)), r_{\alpha,2}(p_2(\sigma_1), \ldots, p_2(\sigma_n)), r_{\alpha,3}(p_3(\sigma_1), \ldots, p_3(\sigma_n)) \rangle$$

[25] For discussions of the general linguistic significance of the distinction, see Barker and Jacobson 2007.

and moreover $\alpha(\sigma_1, \ldots, \sigma_n)$ is defined if and only if each $r_{\alpha,i}$ is defined for the corresponding projections.

As above, however, this is not really a variant of compositionality but rather another way to organize grammars and semantics. This is indicated by (4) and (5) below, which are not hard to verify.[26] First, call G *strict* if whenever $\alpha(\sigma_1, \ldots, \sigma_n)$ is defined and $p_1(\sigma_i) = p_1(\tau_i)$ for $1 \leq i \leq n$, $\alpha(\tau_1, \ldots, \tau_n)$ is defined, and similarly for the other projections. All compositional grammars are strict.

(4) Every grammar G in Kracht's sense for an interpreted language L is a grammar (E, A, Σ) in the sense of section 2 (with $E = L$, $A =$ the set of atomic signs in L, and $\Sigma =$ the set of partial functions of G). Provided G is strict, G is compositional (in Kracht's sense) iff each of p_1, p_2, and p_3, seen as assignments of values to signs (so p_3 is the meaning assignment), is compositional (in our sense).

(5) Conversely, if $\mathbf{E} = (E, A, \Sigma)$ is a grammar and μ a semantics for the grammatical terms of $\mathbf{E}$, let $L = \{\langle u, u, \mu(u)\rangle : u \in \mathit{dom}(\mu)\}$. Define a grammar G for L (with the obvious atomic signs) by letting

$$\alpha(\langle u_1, u_1, \mu(u_1)\rangle, \ldots, \langle u_n, u_n, \mu(u_n)\rangle) =$$
$$\langle \alpha(u_1, \ldots, u_n), \alpha(u_1, \ldots, u_n), \mu(\alpha(u_1, \ldots, u_n))\rangle$$

whenever $\alpha \in \Sigma$ is defined for $u_1, \ldots, u_n$ and $\alpha(u_1, \ldots, u_n) \in \mathit{dom}(\mu)$ (undefined otherwise). Provided μ is closed under subterms and has the Husserl property, μ is compositional iff G is compositional.[27]

3.7 Context-dependence 1 (extra-linguistic context)

In standard possible-worlds semantics the role of meanings are served by the *intensions*, i.e. functions from possible worlds to extensions. For instance, the intension of a sentence s, $\mathbf{I}(s)$ is a function that for a possible world w as argument returns a truth value, if the function is defined for w. Montague

[26] It may seem more natural to extract from G a semantics in the sense of a function from *strings* to meanings, rather than from signs to meanings as in (4). This, however, cannot be done without extra assumptions on G. For example, one might want G to allow for *ambiguity*, i.e. the possibility of $\sigma = \langle e, k, m\rangle$ and $\sigma' = \langle e, k, m'\rangle$ belonging to L while $m \neq m'$; here σ and σ' may be atomic (lexical ambiguity) or even complex with the same derivation history; cf. section 3.4 of Part II. This would be a use of Kracht's format going beyond the organization of grammars and semantics used here, and would exclude a functional assignment of meanings to strings.

[27] Here we have secured strictness of G by letting each term u_i make up its own grammatical category. If the grammar $\mathbf{E}$ is *inductive* in the sense of footnote 8, we can instead more naturally assign categories that correspond to the partition sets.

(1974) extended this idea to include not just worlds but arbitrary *indices* i from some set $\mathbf{I}$, as ordered n-tuples of contextual factors that are relevant to semantic evaluation. Time and place of utterance are typical elements in such indices. The semantic function μ assigns a meaning to a term t, such that $\mu(t)$ itself is a function μ_t that for an index $i \in \mathbf{I}$, $\mu_t(i)$ gives an extension as value.

For such an apparatus, the concept of compositionality can be straightforwardly applied. The situation gets more complicated when the semantic function itself takes contextual arguments, e.g. if a meaning-in-context for a term t in context c is given as $\mu(t, c)$. The reason for such a change might be the view that the contextual meanings are contents in their own right, not just extensional fall-outs of the standing, context-independent meaning. But with context as a separate argument to the semantic function, we have a new source of variation. The most natural extension of compositionality to such a context semantics is given by

C-Funct(μ) For every rule $\alpha \in \Sigma$ there is a meaning operation r_α such that for every context c, if $\alpha(u_1, \ldots, u_n)$ has meaning in c, then $\mu(\alpha(u_1, \ldots, u_n), c) = r_\alpha(\mu(u_1, c), \ldots, \mu(u_n, c))$.[28]

C-Funct(μ) seems like a straightforward extension of compositionality to a contextual semantics, but it can fail in a way non-contextual semantics cannot, by a *context-shift failure*. For we can suppose that although $\mu(u_i, c) = \mu(u_i, c')$, $1 \leq i \leq n$, we still have $\mu(\alpha(u_1, \ldots, u_n), c) \neq \alpha(u_1, \ldots, u_n), c')$. One could claim that this is a possible result of so-called *unarticulated constituents*. Maybe the meaning of the sentence

(6) It rains

is sensitive to the location of utterance, while none of the *constituents* of that sentence (say, *it* and *rains*) is sensitive to location. Then the contextual meaning of the sentence at a location l is different from the contextual meaning of the sentence at another location l', even though there is no such difference in contextual meaning for any of the parts (cf. Perry 1986). This may hold even if substitution of *expressions* is compositional.

There is therefore room for a weaker principle that cannot fail in this way, where the meaning operation *itself* takes a context argument:

[28]Here we have simplified matters by assuming that the extra-linguistic context does not change as evaluation moves to the subterms and between the subterms. This possibility requires a complication of the framework but does not present any problem of principle.

C-Funct$(\mu)_w$ For every rule $\alpha \in \Sigma$ there is a meaning operation r_α such that for every context c, if $\alpha(u_1, \dots, u_n)$ has meaning in c, then $\mu(\alpha(u_1, \dots, u_n), c) = r_\alpha(\mu(u_1, c), \dots, \mu(u_n, c), c)$.

The only difference is the last argument of r_α. Because of this argument, C-Funct$(\mu)_w$ is not sensitive to the counterexample above, and is more similar to non-contextual compositionality in this respect.

Standing meanings can be derived from contextual meanings by abstracting over the context argument: $\mu_s(t) = \lambda c(\mu(t, c))$, where μ_s is the semantic function for standing meaning. It can be shown that if μ obeys C-Funct$(\mu)_w$ or C-Funct(μ), then μ_s obeys Funct(μ). That is, compositionality for contextual meaning entails compositionality for standing meaning. The converse does not hold, for we can let $\mu(t, c) = \mu(u, c)$, while $\mu(t, c') \neq \mu(u, c')$. Then, if $\mu(\alpha(t), c) \neq \mu(\alpha(u), c)$, we have substitution failure in context c for contextual meaning, but t and u cannot yield substitution failure for standing meaning, since their standing meanings are different (see Pagin 2005 and, for a general survey of compositionality issues in connection with (extra-linguistic) context, Westerståhl 2009).

3.8 Context-dependence 2 (linguistic context)

So far, we have been concerned with extra-linguistic context, but we can also extend compositional semantics to dependence on *linguistic context*. That is, the semantic value of some particular *occurrence* of an expression may depend on whether that is an occurrence in, say, an extensional context, or an intensional context, or a hyperintensional context, a quotation context, or yet something else.

A framework for such a semantics needs a finite set C of *context types*, including an initial *null context type* $\theta \in C$ for unembedded occurrences (i.e. terms simpliciter). When a term $\alpha(t_1, \dots, t_n)$ occurs in a context type c, the context types of the t_i may be distinct from c, and thus their semantic contribution may also be distinct from the contribution in c. For example, c can be a quotation context, so that a subterm t_i is mentioned, not used, i.e. the semantic value is the (string value of the) term itself rather than its usual value.

Similarly to C-Funct$(\mu)_w$, the semantic function μ takes a term t and a context type c to a semantic value, the only difference being that in the clause for complex terms, the context types of the subterms may be different, according to this format:

LC-Funct(μ, C) For every rule $\alpha \in \Sigma$ there is an operation r_α such that for any context type $c \in C$ there are $c_1, \ldots, c_n \in C$ such that, if $\alpha(u_1, \ldots, u_n)$ has meaning in c, then

(i) $\mu(\alpha(u_1, \ldots, u_n), c) = r_\alpha(\mu(u_1, c_1), \ldots, \mu(u_n, c_n), c)$

Alternatively, instead of a single $(n+1)$-place semantic function μ taking linguistic context arguments from a finite set C, we can equivalently have a finite set S of n-place semantic functions that includes a designated function μ_θ for unembedded term occurrences. Then the corresponding format is

LC-Funct(S) For every rule $\alpha \in \Sigma$ and semantic function $\mu \in S$ there is an operation $r_{\alpha,\mu}$ and functions $\mu_1, \ldots, \mu_n \in S$ such that if $\alpha_i(u_1, \ldots, u_n)$ has μ-meaning, then

(ii) $\mu(\alpha(u_1, \ldots, u_n)) = r_{\alpha,\mu}(\mu_1(u_1), \ldots, \mu_n(u_n))$

In this set-up, the *semantics* is the whole set S of functions assigning semantic values to terms. It is easy to verify that LC-Funct(μ, C) and LC-Funct(S) are equivalent generalizations of standard compositionality.[29] The generalized versions are considerably more powerful for handling special contexts. This will be exemplified with a semantics for quotation contexts in section 3.2 of Part II.

References

Abelard, Peter (2008). *Logica 'ingredientibus'. 3. Commentary on Aristotele's* De Interpretatione. Ed. by Klaus Jacobi and Christian Straub. Corpus Christianorum Contiunatio Medievalis. Turnhout: Brepols Publishers.

Ailly, Peter of (1980). *Concepts and Insolubles.* Ed. by Paul Vincent Spade. Original published as *Conceptus et Insolubilia*, Paris, 1500. Dordrecht: Reidel.

Barker, Chris and Pauline Jacobson, eds. (2007). *Direct Compositionality.* Oxford: Oxford University Press.

Buridan, John (1998). *Summulae de Dialectica 4. Summulae de Suppositionibus.* Ed. by Ria van der Lecq. Vol. 10-4. Artistarium. Nijmegen.

[29] More precisely, if LC-Funct(μ, C) holds, define, for $c \in C$, $\mu_c(t) = \mu(t, c)$, and let $S = \{\mu_c : c \in C\}$. Then LC-Funct(S) holds with μ_θ as the designated semantic function.

Conversely, suppose LC-Funct(S) holds. Define a semantics ν by letting, for all $\mu \in S$ and all terms t, $\nu(t, \mu) = \mu(t)$ (when the latter is defined). That is, use the functions in S themselves as context types. Then one can verify that LC-Funct(ν, S) holds, with μ_θ as the designated context type.

The generalized versions have standard compositionality as a special case: LC-Funct$(\{\mu\})$ is equivalent to Funct(μ), and the same holds for LC-Funct$(\mu, \{c\})$

Carnap, Rudolf (1956). *Meaning and Necessity.* 2nd ed. Chicago, Ill.: The University of Chicago Press.

Dowty, David (2007). 'Compositionality as an Empirical Problem'. In: *Compositionality as an Empirical Problem.* Ed. by Chris Barker and Pauline Jacobson. Oxford: Oxford University Press, pp. 23–101.

Dummett, Michael (1973). *Frege. Philosophy of Language.* London: Duckworth.

— (1981). *The Interpretation of Frege's Philosophy.* London: Duckworth.

Fernando, Tim (2005). 'Compositionality inductively, co-inductively and contextually'. In: *The Compositionality of Meaning and Content: Foundational Issues.* Ed. by Edouard Machery, Markus Werning, and Gerhard Schurz. Vol. I. Frankfurt/Lancaster: Ontos, pp. 87–96.

Fodor, Jerry (1987). *Psychosemantics.* Cambridge, Mass.: MIT Press.

— (2000). 'Reply to critics'. In: *Mind & Language* 15, pp. 350–374.

— (2008). *LOT2.* Oxford: Oxford University Press.

Fodor, Jerry and Jerrold Katz (1964). 'The Structure of a Semantic Theory'. In: *The Structure of Language.* Ed. by Jerry Fodor and Jerrold Katz. Englewood Cliffs, NJ.: Prentice-Hall, pp. 479–518.

Frege, Gottlob (1884). *Grundlagen der Arithmetik,* translated by J. Austin as *Foundations of Arithmetic,* 1950. Oxford: Blackwell.

— (1892). 'Über Sinn und Bedeutung'. In: *Translations from the Philosophical Writings of Gottlob Frege,* 1952. Ed. by Peter Geach and Max Black. Oxford: Blackwell, pp. 56–78.

Hendriks, Herman (2001). 'Compositionality and model-theoretic interpretation'. In: *Journal of Logic, Language and Information* 10, pp. 29–48.

Hinzen, Wolfram, Edouard Machery, and Markus Werning, eds. (2009). *The Oxford Handbook of Compositionality.* Oxford: Oxford University Press.

Hodges, Wilfrid (2001). 'Formal features of compositionality'. In: *Journal of Logic, Language and Information* 10, pp. 7–28.

Houben, Jan (1997). 'The Sanskrit Tradition'. In: *The Sanskrit Tradition.* Ed. by Wout van Bekkum et al. Vol. 82. Amsterdam Studies in the Theory and History of Linguistic Science. Amsterdam, Philadelphia: John Benjamin Publishing, pp. 49–145.

Husserl, Edmund (1900). *Logische Untersuchungen II/1,* translated by J. N. Findlay as *Logical Investigations,* 1970. London and Henley: Routledge and Kegan Paul.

Jacobson, Pauline (2002). 'The (Dis)organisation of the Grammar: 25 years'. In: *Linguistics & Philosophy* 25, pp. 601–26.

Janssen, Theo (1986). *Foundations and Applications of Montague Grammar.* Amsterdam: CWI Tracts 19 and 28.

— (1997). 'Compositionality'. In: *Handbook of Logic and Language.* Ed. by Johan van Benthem and Alice ter Meulen. Amsterdam: Elsevier, pp. 417–73.

— (2001). 'Frege, contextuality and compositionality'. In: *Journal of Logic, Language and Information* 10, pp. 115–136.

Keenan, Edward and Edward Stabler (2004). *Bare Grammar: A Study of Language Invariants.* Stanford: CSLI Publications.

King, Peter (2001). 'Between Logic and Psychology. John Buridan on Mental Language'. Presented at the conference *John Buridan and Beyond*, Copenhagen, September 2001.

— (2007). 'Abelard on Mental'. In: *American Catholic Philosophical Quarterly* 81, pp. 169–87.

Kracht, Marcus (2003). *The Mathematics of Language.* Berlin: Mouton de Gruyter.

— (2007). 'The emergence of syntactic structure'. In: *Linguistics & Philosophy* 30, pp. 47–95.

Larson, Richard and Gabriel Segal (1995). *Knowledge of Meaning. An Introduction to Semantic Theory.* Cambridge, Mass.: MIT Press.

Machery, Edouard, Markus Werning, and Gerhard Schurz, eds. (2005). *The Compositionality of Meaning and Content: Foundational Issues.* Vol. I. Frankfurt/Lancaster: Ontos.

Montague, Richard (1974*a*). *Formal Philosophy* (edited and with an introduction by Richmond Thomason). New Haven: Yale University Press.

— (1974*b*). 'Pragmatics'. In: *Formal Philosophy* (edited and with an introduction by Richmond Thomason). Originally published in Raymond Klibansky (ed.), *Contemporary Philosophy: A Survey*, La Nuova Italia Editrice, Florence, 1986, pp. 102-22. New Haven: Yale University Press, pp. 95–118.

Pagin, Peter (2003). 'Communication and strong compositionality'. In: *Journal of Philosophical Logic* 32, pp. 287–322.

— (2005). 'Compositionality and context'. In: *Contextualism in Philosophy.* Ed. by Gerhard Preyer and Georg Peter. Oxford: Oxford University Press, pp. 303–48.

— (2009). 'Communication and the Complexity of Semantics'. In: *The Oxford Handbook of Compositionality.* Ed. by Wolfram Hinzen, Edouard Machery, and Markus Werning. Oxford: Oxford University Press.

Pelletier, Francis Jeffry (2001). 'Did Frege believe Frege's Principle?' In: *Journal of Logic, Language and Information* 10, pp. 87–114.

Perry, John (1986). 'Thought without representation'. In: *Proceedings of the Aristotelian Society* 60, pp. 263–83.

Peters, Stanley and Dag Westerståhl (2006). *Quantifiers in Language and Logic.* Oxford: Oxford University Press.

Potts, Christopher (2007). 'The Dimensions of Quotation'. In: *Direct Compositionality.* Ed. by Chris Barker and Pauline Jacobson. Oxford: Oxford University Press, pp. 405–31.

Szabó, Zoltán (2000). 'Compositionality as Supervenience'. In: *Linguistics & Philosophy* 23, pp. 475–505.

— (2007). 'Compositionality'. In: *Stanford Encyclopedia of Philosophy.* Ed. by Edward N. Zalta. Stanford University. URL: <http://plato.stanford.edu/archives/spr2007/entries/compositionality/>.

Werning, Markus (2005*a*). 'Neural Synchronization, Covariation, and Compositional Representation'. In: *The Compositionality of Meaning and Content: Application to Linguistics, Psychology, and Neuroscience.* Ed. by Edouard Machery, Markus Werning, and Gerhard Schurz. Vol. II. Ontos, pp. 283–312.

— (2005*b*). 'Right and Wrong Reasons for Compositionality'. In: *The Compositionality of Meaning and Content: Foundational Issues.* Ed. by Edouard Machery, Markus Werning, and Gerhard Schurz. Vol. I. Frankfurt/Lancaster: Ontos, pp. 285–309.

Westerståhl, Dag (2004). 'On the compositional extension problem'. In: *Journal of Philosophical Logic* 33, pp. 549–582.

— (2008). 'Decomposing generalized quantifiers'. In: *Review of Symbolic Logic* 1:3, pp. 355–371.

— (2009). 'Compositionality in Kaplan style semantics'. In: *The Oxford Handbook of Compositionality.* Ed. by Wolfram Hinzen, Edouard Machery, and Markus Werning. Oxford: Oxford University Press.