Priority Structure in Deontic Logic

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Joint project with van Benthem and Grossi

- Johan van Benthem, Davide Grossi, and Fenrong Liu: Deontics = Betterness + Priority, in Guido Governatori and Giovanni Sartor eds. *Deontic Logic in Computer Science*, 10th International Conference, DEON 2010, Fiesole, Italy, July 7-9, 2010. Proceedings. *Lecture Notes in Artificial Intelligence*, Vol. 6181, Springer. pp.50-65.
- Johan van Benthem, Davide Grossi , and Fenrong Liu: Priority Structures in Deontic Logic, under submission, 2012.

Outline

Ideality ordering in deontic logic

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- Main operators: $O\varphi$(obligation), $P\varphi$(permission), and $O(\varphi \mid \psi)$.
- Long-standing meta-ethical intuition: these notions involve a normative "ideality ordering".

"[. . .] to assert that a certain line of conduct is, at a given time, absolutely right or obligatory, is obviously to assert that more good or less evil will exist in the world, if it is adopted, than if anything else be done instead" [Moore, Principia Ethica, 1922]

Deontic logic models and betterness relation

Deontic logic models involving betterness ordering of worlds or states. [Hansson, 1969]

Conditional obligation

$\mathbf{O}(\varphi \mid \psi)$ were interpreted in terms of a binary relation $s \preceq t$ between states s, t:

$$\mathcal{M}, s \models \mathbf{O}(\varphi \mid \psi) \Longleftrightarrow \max_{\preceq}([[\psi]]_{\mathcal{M}}) \subseteq [[\varphi]]_{\mathcal{M}} \tag{1}$$

max is a 'selection function' picking out the $\preceq$-maximal elements in any given set.

Reasons for betterness ordering

The betterness relation between situations that grounds our obligations (or preference in general) often stems from an explicit code for what is right or wrong.

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We call structures of this kind "priority sequences" or "priority graphs" (when it is a partial order).

Priority graphs

Definition (P-graphs)

Let $\mathcal{L}(\mathbf{P})$ be a propositional language built on the set of atoms $\mathbf{P}$. A P-graph is a tuple $\mathcal{G} = \langle \Phi, \prec \rangle$ such that:

- $\Phi \subset \mathcal{L}(\mathbf{P})$ with $|\Phi| < \omega$;
- $\prec$ is a strict order on Φ : property ψ is strictly better than φ; also,
  for all propositions $\varphi, \psi \in \Phi$: if $\varphi \prec \psi$ then φ logically implies ψ.

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Intuitively, a P-graph is a finite graph of formulae from a propositional language, where each formula logically implies its immediate successor in the order

State betterness from P-graphs

Definition (State betterness from P-graphs)

Let $\mathcal{G} = \langle \Phi, \prec \rangle$ be a P-graph, S a non-empty set of states and $\mathcal{I} : \mathbf{P} \longrightarrow 2^S$ a valuation function. The betterness relation $\preceq_{\mathcal{G}} \subseteq S^2$ is defined as follows:

$$s \preceq_{\mathcal{G}} s' \quad := \quad \forall \varphi \in \Phi : s \in [[\varphi]]_{\mathcal{I}} \Rightarrow s' \in [[\varphi]]_{\mathcal{I}}. \tag{3}$$

The function outputting this pre-order is called `sub` (from‘subsumption’).

Here are some useful properties:

Fact (Basic properties of $\preceq_{\mathcal{G}}$)

Let $\mathcal{G} = \langle \Phi, \prec \rangle$ be a P-graph. For any valuation $\mathcal{I} : \mathbf{P} \longrightarrow 2^S$ it holds that:

1. *The relation $\preceq_{\mathcal{G}}$ is a pre-order whose strict part $\prec_{\mathcal{G}}$ is upward well-founded;*
2. *If $\varphi_i \prec \varphi_j$, then for all worlds $s \in \varphi_{i\mathcal{I}}$, $s' \in \varphi_{j\mathcal{I}}$: $s \preceq_{\mathcal{G}} s'$;*
3. *If $\varphi_i \prec \varphi_j$, then for all worlds $s \in \varphi_i \wedge \neg\varphi_{j\mathcal{I}}$, $s' \in \varphi_{j\mathcal{I}}$: $s \prec_{\mathcal{G}} s'$.*

A logic of betterness

The basic modal language of preference $\mathcal{L}(\mathsf{U}, \preceq)$ is built from a countable set **P** of atoms according to the following inductive syntax:

$$\mathcal{L}(\mathsf{U}, \preceq) : \varphi ::= p \mid \top \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\preceq]\varphi \mid [\mathsf{U}]\varphi$$

Models

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The strict suborder $\prec$ (“strictly better than") of $\preceq$ as usual: $s \prec s'$ iff $s \preceq s'$ and $s' \not\preceq s$.

Definition (Truth condition)

Let $\mathcal{M} = \langle S, \preceq, \mathcal{I} \rangle$ be a model. Truth for a formula $\varphi \in \mathcal{L}(\mathsf{U}, \preceq)$ in a pointed model $(\mathcal{M}, s)$ is defined inductively:

$$
\begin{array}{rcl}
\mathcal{M}, s \models p & \iff & w \in \mathcal{I}(p) \\
\mathcal{M}, s \models [\preceq]\varphi & \iff & \forall s' \in S \text{ s.t. } s \preceq s' : \mathcal{M}, s' \models \varphi \\
\mathcal{M}, s \models [\mathsf{U}]\varphi & \iff & \forall s' \in S : \mathcal{M}, s' \models \varphi
\end{array}
$$

Axiomatization

A complete axiomatic proof calculus for our system consists of the standard modal logic **S4** for betterness, **S5** axioms for the universal modality, and one inclusion axiom **Incl**$([U]\varphi \to [\preceq]\varphi)$. This logic is known to be sound and strongly complete for pre-orders. Its uses go back to [Boutilier, 1994].

Models from P-graphs

Given a P-graph $\mathcal{G}$ and a propositional valuation function $\mathcal{I}$, we can obtain models of the above type $\mathcal{M} = \langle S, \preceq_{\mathcal{G}}, \mathcal{I}\rangle$ where $\preceq_{\mathcal{G}}$ is the total pre-order derived from $\mathcal{G}$.

Expressive power: defining semantic 'best'.

Our modal language can define various notions:

Fact

On total pre-orders $\preceq$ with an upward well-founded strict part $\prec$, conditional obligation in the sense of [Hansson, 1969] is defined as follows:

$$\mathbf{O}_{\preceq}(\varphi \mid \psi) \quad := \quad [\mathsf{U}](\psi \rightarrow \langle\preceq\rangle(\psi \wedge [\preceq](\psi \rightarrow \varphi))) \tag{4}$$

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This equivalence may be understood as follows. On well-founded ordering models, all maximal states satisfy φ iff, for every world, there is a better world all of whose better alternatives satisfy φ, i.e., $[U]\langle\preceq\rangle[\preceq]\varphi$. Relativizing in the usual manner to a formula ψ yields Formula (4).

Adding priorities: fitting in syntactic 'best'

- The definition of maximality in Formula (4) relies on an underlying order.
- In terms of P-graphs $\mathcal{G}$, the matching intuition is: the best states under condition ψ are those that belong to the most ideal properties in the P-graph compatible with ψ, in the sense of non-empty intersection of their truth-sets.
- Call this more syntactic notion $\mathbf{O}_{\mathcal{G}}(. \mid .)$. It says: 'the best properties of $\mathcal{G}$ that are compatible with ψ all imply φ'.

Given a graph $\mathcal{G}$, this is definable in our modal language:

$$\mathbf{O}_{\mathcal{G}}(\varphi \mid \psi) \quad := \quad [\mathsf{U}]\left(\left(\bigvee_{\langle\varphi_1,\ldots,\varphi_n\rangle \in S_{\mathcal{G}}} \bigwedge_{1 \leq i \leq n} (\langle \mathsf{U}\rangle(\varphi_i \wedge \psi) \to (\varphi_i \wedge \psi))\right) \to \varphi\right) \tag{5}$$

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Here, for $\langle \varphi_1, \ldots, \varphi_n\rangle \in S_{\mathcal{G}}$, $\bigwedge_{1\leq i\leq n} (\langle \mathsf{U}\rangle(\varphi_i \wedge \psi) \rightarrow (\varphi_i \wedge \psi))$ denotes the maximal element of the P-sequence $\langle \varphi_1, \ldots, \varphi_n\rangle$ that has a non-empty intersection with ψ. Then $\bigvee_{\langle \varphi_1,\ldots,\varphi_n\rangle \in S_{\mathcal{G}}} \bigwedge_{1\leq i\leq n} (\langle \mathsf{U}\rangle(\varphi_i \wedge \psi) \rightarrow (\varphi_i \wedge \psi))$ takes the union of these, and Formula (5) states that this is subsumed by φ.

Correspondence of syntactic and semantic 'best'.

Theorem (Correspondence)

Let $\mathcal{G} = \langle \Phi, \prec \rangle$ be a P-graph, $\mathcal{M}_{\mathcal{G}}$ a model derived by Definition 2 from $\mathcal{G}$, $\mathcal{I}$ a valuation function and s a state:

$$\mathcal{M}_{\mathcal{G}}, s \models \mathbf{O}_{\preceq}(\varphi \mid \psi) \iff \mathcal{M}_{\mathcal{G}}, s \models \mathbf{O}_{\mathcal{G}}(\varphi \mid \psi).$$

Introducing priorities

J. Forrester. Gentle murder, or the adverbial samaritan. Journal of Philosophy, 81:193-197, 1984.

Example (Gentle murder)

“Let us suppose a legal system which forbids all kinds of murder, but which considers murdering violently to be a worse crime than murdering gently. [. . .] The system then captures its views about murder by means of a number of rules, including these two:

1. It is obligatory under the law that Smith not murder Jones.
2. It is obligatory that, if Smith murders Jones, Smith murders Jones gently."

Gentle murder

The scenario mentions two classes of states: those in which Smith does not murder Jones($\neg m$); and those in which either Smith does not murder Jones or he does murders Jones, but gently($\neg m \vee (m \wedge g)$. The P-sequence is $(\neg m \vee (m \wedge g)) \prec \neg m$. The induced betterness relation orders worlds in three disjoint clusters, shown below:

$\neg m$	$m \wedge g$	$m \wedge \neg g$

The Chisholm scenario

Example (The Chisholm scenario)

1. It ought to be that Smith refrains from robbing Jones.
2. Smith robs Jones.
3. If Smith robs Jones, he ought to be punished for robbery.
4. It ought to be that if Smith refrains from robbing Jones he is not punished for robbery.

Again, three items specify a priority sequence: $\neg r \vee (r \wedge p) \prec (\neg r \wedge \neg p)$ (where r stands for "Smith robs Jones" and p for "Smith is punished").

CTDs and priority structures

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- CTDs has been used as benchmarks to evaluate logics of conditional obligation.
- P-graphs is the natural formalization of a CTD: some norms are given, and obligations are computed going 'down the line'.
- We have reasons available for the betterness ordering.

The Kanger-Anderson reduction and P-sequence

Anderson [Anderson, 1957] and Kanger [Kanger, 1971] reduced deontic **O**-formulae to alethic modal $\Box$-formulae with a constant for violation V or ideality I:

$$\mathbf{O}\varphi := \Box(\neg\varphi \to \mathsf{V}) \tag{6}$$

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We will show how P-graphs, properly specialized, offer a natural extension to Anderson's and Kanger's proposals that does deal with CTDs along the lines they advocated.

Definition (KA-sequences)

Let $\{\mathsf{I}_1, \ldots, \mathsf{I}_n\} \subseteq \mathbf{P}$. A Kanger-Anderson sequence ("KA-sequence") for $\mathcal{L}(\mathbf{P})$ is a sequence defined as follows:

$$\left\langle \bigvee_{1 \leq j \leq i} \mathsf{I}_j \right\rangle_{1 \leq i \leq n}$$

KA-sequences are tuples $\langle \mathsf{I}_1, \mathsf{I}_1 \vee \mathsf{I}_2, \ldots, \mathsf{I}_1 \vee \ldots \vee \mathsf{I}_n \rangle$ which are built by using ideality atoms to construct n layers spanning from the most to the least ideal.

The Correspondence Theorem specializes to KA-sequences in an interesting way:

Corollary (Obligations from better to worse)

Let $\mathcal{G}$ be a KA-sequence. For any model $\mathcal{M}_{\mathcal{G}}$, state s, and $1 \leq i < n$ it holds that:

$$\mathcal{M}_{\mathcal{G}}, s \models \mathbf{O}(\varphi \mid \top) \iff \mathcal{M}_{\mathcal{G}}, s \models [\mathsf{U}](\mathsf{I}_1 \to \varphi) \tag{8}$$

$$\mathcal{M}_{\mathcal{G}}, s \models \mathbf{O}(\varphi \mid \mathsf{I}_1) \iff \mathcal{M}_{\mathcal{G}}, s \models [\mathsf{U}](\mathsf{I}_1 \to \varphi) \tag{9}$$

$$\mathcal{M}_{\mathcal{G}}, s \models \mathbf{O}(\varphi \mid \neg \bigvee_{1 \leq j \leq i} \mathsf{I}_j) \iff \mathcal{M}_{\mathcal{G}}, s \models [\mathsf{U}](\bigvee_{1 \leq j \leq i+1} \mathsf{I}_j \to \varphi) \tag{10}$$

$$\mathcal{M}_{\mathcal{G}}, s \models \mathbf{O}(\varphi \mid \neg \bigvee_{1 \leq j \leq n} \mathsf{I}_j) \iff \mathcal{M}_{\mathcal{G}}, s \models [\mathsf{U}](\neg \bigvee_{1 \leq j \leq n} \mathsf{I}_j \to \varphi) \tag{11}$$

- Formula 8 says that an unconditional obligation $\mathbf{O}(\varphi \mid \top)$ is what the most ideal states dictate.
- The corollary shows how obligations changes as we move from most to least ideal circumstances. In most ideal states, where I_1 holds, what ought to be the case is what already is the case (Formula 9).
- Formula 10 states that, if the i^{th} element has been violated, what ought to be is what follows from the $(i+1)^{th}$ element in the sequence.
- And in the least ideal states, where I_n is false, what ought to be the case is again what is already the case (Formula 11). This generalizes the Anderson-Kanger reduction to CTD reasoning.

Recall...

$$\mathcal{M}, s \models [!\varphi]\psi \iff \text{if } \mathcal{M}, s \models \varphi \text{ then } \mathcal{M}|\varphi, s \models \psi. \quad (12)$$

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$$[!\varphi]\,\mathbf{O}(\psi \mid \chi) \leftrightarrow (\varphi \to \mathbf{O}([!\varphi]\psi \mid (\varphi \wedge [!\varphi]\chi)) \quad (14)$$

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$$[!\varphi]\mathbf{O}(\psi \mid \chi) \leftrightarrow (\varphi \to \mathbf{O}([!\varphi]\psi \mid (\varphi \wedge [!\varphi]\chi)) \quad (14)$$

$$[!\varphi]\mathbf{O}\psi \leftrightarrow (\varphi \to \mathbf{O}([!\varphi]\psi \mid \varphi)) \quad (15)$$

Information dynamics in conditional obligations

- Obligations are typically conditional, so changes in circumstances determine changes in what ought to be the case.
- Semantically, this means that maximally ideal states change under different circumstances.
- Syntactically, this means that properties in the priority structure that are incompatible with current circumstances can be disregarded. This is often called *deontic detachment*: conditional obligations remain stable, but what changes is what follows from them under different circumstances.
- We will show that our structured models naturally give us these two faces of deontic dynamics.

The Chisholm scenario: a dynamic perspective

Example (The Chisholm scenario: a dynamic perspective)

The Chisholm scenario consisted of three normative statements:

- It ought to be that Smith refrains from robbing Jones.
- If Smith robs Jones, he ought to be punished for robbery.
- It ought to be that if Smith refrains from robbing Jones he is not punished for robbery.

plus a factual one:

- Smith robs Jones.

- There is a difference in function here. The normative statements seem global guides to behavior, but the scenario suggests a dynamic reading of the factual statement.

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An illustration is the following dynamic formula matching Example 9):

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if it ought to be the case that $\neg r$, that p if r, then, after it is learnt that r is the case, it ought to be the case that p.
The formula is true in all betterness **S4** models derived by the priority sequence of the Chisholm scenario.

Formula 16 is a special case of the following validity of our logic, that holds for factual formulas:

$$\mathbf{O}(\varphi \mid \psi) \rightarrow [!\psi]([\mathsf{U}]\psi \wedge \mathbf{O}(\varphi \mid \top))^{1} \tag{17}$$

Reading universal modality [U] as an epistemic operator, formula 17 formalizes the principle: "If it ought to be the case that φ under condition that ψ then, if it is announced that ψ is the case, it is known that ψ and it ought to be (unconditionally) the case that φ".

[1] The restriction to factual formulas is useful, but not essential: cf. [Liu, 2011].

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This suggests a natural form of conditional obligations taken in an epistemic sense: I should do something if I *know* the antecedent to be the case. C.f. a nice example in [Pacuit et al., 2006].

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P-graph restriction

Definition (P-graph restriction)

Let $\mathcal{G} = \langle \Phi, \prec \rangle$ be a P-graph, and ψ a formula. The restriction of $\mathcal{G}$ by ψ is the graph $\mathcal{G}^{\psi} = \langle \Phi^{\psi}, \prec^{\psi} \rangle$ where:

- $\Phi^{\psi} = \{\varphi \wedge \psi \mid \varphi \in \Phi\}$;
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The restriction of a P-graph $\mathcal{G}$ by ψ simply intersects the elements of the original graph with ψ and keeps the original order.

Theorem (Harmony of P-graph restriction)

The following diagram commutes for all P-graphs $\mathcal{G}$, propositional formula φ and valuation $\mathcal{I}$:

$$\begin{array}{ccc} \mathcal{G} & \longrightarrow & \mathcal{G}^{\psi} \\ \mathtt{sub}\downarrow & & \downarrow\mathtt{sub} \\ \langle S, \preceq_{\mathcal{G}}, \mathcal{I}\rangle & \xrightarrow{!\psi} & \langle \psi, \preceq_{\mathcal{G}^{\psi}}, \mathcal{I}|_{\psi}\rangle \end{array}$$

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Postfixing norms

Example (Postfixing norms)

For instance, start the earlier Gentle Murder scenario with the P-sequence

$$\langle \neg m \rangle$$

This generates a total pre-order with all $\neg m$ states above all m states: “It is obligatory under the law that Smith not murder Jones".

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Now, a lawgiver with moral authority comes in, and introduces the sub-ideal obligation to kill gently: “it is obligatory that, if Smith murders Jones, Smith murders Jones gently." This can be done by postfixing the original sequence with the property $\neg m \vee g$:

$$\langle \neg m, \neg m \vee g \rangle$$

Prefixing norms

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Recall the P-sequence of the gentle murder case:

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Prefixing norms

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Recall the P-sequence of the gentle murder case:

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Now we want to introduce a stronger norm, like "It is obligatory under the law that Smith not murder Jones and that Smith not be aggressive against Jones". This can be achieved syntactically by prefixing the P-sequence with $\neg m \wedge \neg a$, where a stands for "Smith is aggressive against Jones":

$$\langle (\neg m \wedge \neg a), \neg m, \neg m \vee g \rangle$$

Prefixing and postfixing in P-graphs

Definition (Prefixing and postfixing in P-graphs)

Let $\mathcal{G} = \langle \Phi, \prec \rangle$ be a P-graph, and φ a propositional formula:

- the prefixing of $\mathcal{G}$ by φ yields the graph $\varphi; \mathcal{G}$ where a new maximal element $\varphi \wedge \bigwedge \max(\mathcal{G})$ is added to $\mathcal{G}$, consisting of the conjunction of φ with the conjunction of the maximal of $\mathcal{G}$;
- the postfixing of $\mathcal{G}$ by φ yields the graph $\mathcal{G}; \varphi$ where a new minimal element $\varphi \vee \bigvee \min(\mathcal{G})$ is added to $\mathcal{G}$, consisting of the disjunction of φ with the disjunction of the minimal elements of $\mathcal{G}$.

Theorem: Harmony of P-graph pre-\post-fixing

The following diagram commutes for all P-graphs $\mathcal{G}$, propositional formulae φ and valuations $\mathcal{I}$:

$$\begin{tikzcd} \mathcal{G} \arrow[r, "\star\varphi"] \arrow[d, "\mathtt{sub}"'] & \mathcal{G} \star \varphi \arrow[d, "\mathtt{sub}"'] \\ \langle S, \preceq_{\mathcal{G}}, \mathcal{I} \rangle \arrow[r, "\Uparrow f_\star(\varphi)"] & \langle S, \preceq_{\mathcal{G}\star\varphi}, \mathcal{I} \rangle \end{tikzcd}$$

where $\mathcal{G} \star \varphi$ denotes either the pre-fixing $\varphi; \mathcal{G}$ or the post-fixing $\mathcal{G}; \varphi$ of $\mathcal{G}$ by φ and $f_\star(\varphi)$ denotes accordingly $\varphi \wedge \bigwedge \max(\mathcal{G})$ or $\varphi \vee \bigvee \min(\mathcal{G})$.

Example (Quick murder)

Let us assume now there are two normative sources.
According to the first one:

1. It is obligatory under the law that Smith not murder Jones.
2. It is obligatory that, if Smith murders Jones, Smith murders Jones gently."

According to the second one:

1. It is obligatory under the law that Smith not murder Jones.
2. It is obligatory that, if Smith murders Jones, Smith murders Jones *quickly*."

Paralell composition

We can model this scenario by means of two P-sequences

- $\mathcal{S}_g$ with $\neg m \succ \neg m \vee (m \wedge g)(= m \rightarrow g)$,
- $\mathcal{S}_q$ with $\neg m \succ \neg m \vee (m \wedge q)(= m \rightarrow q)$.

Paralell composition

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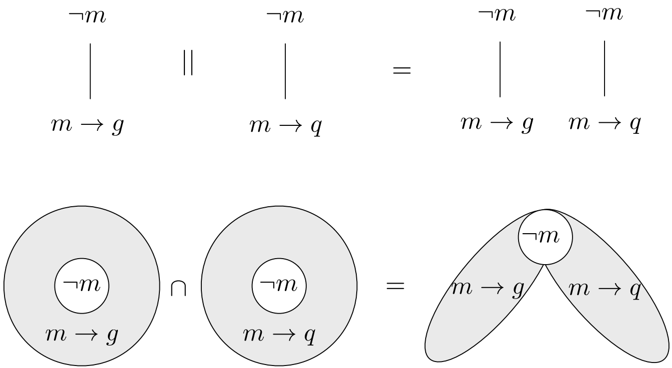
- $\mathcal{S}_g$ with $\neg m \succ \neg m \vee (m \wedge g)(= m \to g)$,
- $\mathcal{S}_q$ with $\neg m \succ \neg m \vee (m \wedge q)(= m \to q)$.

Parallel composition("||"): taking the disjoint union of $\mathcal{S}_g$ and $\mathcal{S}_q$. The resulting pre-order should be such that the best states are $\neg m$-states and the sub-ideal states are split into two incomparable classes, the class of $m \wedge g$-states and the class of $m \wedge q$-states.

Parallel composition in Figure



Harmony of parallel composition of P-graphs

Fact (Harmony of parallel composition of P-graphs)

Let $\mathcal{G} = \langle G, \prec \rangle$ and $\mathcal{G}' = \langle G', \prec' \rangle$ be two P-graphs. The following diagram commutes:

$$
\begin{tikzcd}
\mathcal{G} \arrow[r, "||\mathcal{G}'"] \arrow[d, "\mathtt{sub}"'] & \mathcal{G}||\mathcal{G}' \arrow[d, "\mathtt{sub}"'] \\
\langle S, \preceq_{\mathcal{G}} \rangle \arrow[r, "\cap\ \preceq_{\mathcal{G}'}"] & \langle S, \preceq_{\mathcal{G}||\mathcal{G}'} \rangle
\end{tikzcd}
$$

Summary

- We have shown how deontic scenarios can be mined for more structure than just deontic inferences. Nnormative structure can be represented in priority graphs; informational and deontic events change our current obligations.

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- The new framework brings many relevant new phenomena into the scope of deontic logic, such as norm change and general calculus of normative code.

Further problems

- Extending our analysis from obligations in terms of propositions to obligations among *actions*.
- Connect our dynamics of local informational or deontic events to long term deontic phenomena in agency over time.
- Exploring detailed legal argumentation as a natural test for the richer deontic modeling apparatus proposed here and developing our graph calculus to deal with a richer repertoire of natural operations of norm merge and construction of moral codes.

The End

Thanks!

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Anderson, A. (1957).
The formal analysis of normative concepts.
American Sociological Review, 22:9–17.

Hansson, B. (1969).
An analysis of some deontic logics.
Nous, 3:373–398.

Kanger, S. (1971).
New fondations for ethical theory.
In Hilpinen, R., editor, *Deontic Logic: Introductory and Systematic Readings*, pages 36–58. Reidel Publishing Company.

Liu, F. (2011).
Reasoning about Preference Dynamics, volume 354 of *Synthese Library*.
Springer.

Pacuit, E., Parikh, R., and Cogan, E. (2006).
The logic of knowledge based on obligation.
Synthese, 149:311–341.