Chapter 3
Solved Problems

1. For the function \( y = x^2 - \frac{x}{x+3} \), calculate the value of \( y \) for the following values of \( x \) using element-by-element operations: 0, 1, 2, 3, 4, 5, 6, 7.

Solution

\[
\begin{align*}
\text{>> } & \text{x=1:7} \\
& \text{x =} \\
& \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7
\end{bmatrix}
\end{align*}
\]

\[
\text{>> } y=x.\text{^2-x.}./(x+3)
\]

\[
\begin{bmatrix}
0.7500 & 3.6000 & 8.5000 & 15.4286 & 24.3750 & 35.3333 & 48.3000
\end{bmatrix}
\]

2. For the function \( y = x^4 e^{-x} \), calculate the value of \( y \) for the following values of \( x \) using element-by-element operations: 1.5, 2, 2.5, 3, 3.5, 4.

Solution

\[
\begin{align*}
\text{>> } & \text{x=1.5:0.5:4} \\
& \text{x =} \\
& \begin{bmatrix}
1.5000 & 2.0000 & 2.5000 & 3.0000 & 3.5000 & 4.0000
\end{bmatrix}
\end{align*}
\]

\[
\text{>> } y=x.\text{^4.*exp(-x)}
\]

\[
\begin{bmatrix}
1.1296 & 2.1654 & 3.2064 & 4.0328 & 4.5315 & 4.6888
\end{bmatrix}
\]
3. For the function \( y = (x + \sqrt{x + 3})(1 + 2x^2) - x^3 \), calculate the value of \( y \) for the following values of \( x \) using element-by-element operations: \(-2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2\).

**Solution**

\[
\begin{align*}
\text{x} &= -2:0.5:2 \\
\text{x} &= \begin{bmatrix} -2.0000 & -1.5000 & -1.0000 & -0.5000 & 0 \\ 0.5000 & 1.0000 & 1.5000 & 2.0000 \end{bmatrix} \\
\text{y} &= (x+x.*\sqrt{x+3}).*(1+2*x.^2)-x.^3 \\
\text{y} &= \begin{bmatrix} -28.0000 & -14.9791 & -6.2426 & -1.8109 & 0 \\ 2.0281 & 8.0000 & 22.3759 & 50.2492 \end{bmatrix}
\end{align*}
\]

4. For the function \( y = \frac{4\sin x + 6}{(\cos^2 x + \sin x)^2} \), calculate the value of \( y \) for the following values of \( x \) using element-by-element operations: \(15^\circ, 25^\circ, 35^\circ, 45^\circ, 55^\circ, 65^\circ\).

**Solution**

\[
\begin{align*}
\text{x} &= 15:10:65 \\
\text{x} &= \begin{bmatrix} 15 & 25 & 35 & 45 & 55 & 65 \end{bmatrix} \\
\text{y} &= (4*\sin(x)+6)./(\cosd(x).^2+\sind(x)).^2 \\
\text{y} &= \begin{bmatrix} 4.9528 & 4.9694 & 5.3546 & 6.0589 & 7.0372 & 8.1775 \end{bmatrix}
\end{align*}
\]

5. The radius, \( r \), of a sphere can be calculated from its volume, \( V \), by:

\[
 r = \sqrt[3]{\frac{3V}{4\pi}}
\]

The surface area of a sphere, \( S \), is given by:

\[
 S = 4\pi r^2
\]

Determine the radius and surface area of spheres with volumes of 4,000, 3,500, 3,000, 2,500, 2,000, 1,500 and 1,000 \( \text{in}^3 \). Display the results in a three-column table where the values of \( r \), \( V \), and \( S \) are displayed in the first, second, and third columns, respectively. The values of \( r \) and \( S \) that are displayed in the table should be rounded to the nearest tenth of an inch.

**Solution**
Script File:
V=4000:-500:1000;
r=(3/(4*pi)*V).^(1/3);
S=4*pi*r.^2;
Result(:,1)=round(r*10)/10;
Result(:,2)=V;
Result(:,3)=round(S*10)/10

Command Window:
Result =
9.80 4000.00 1218.60
9.40 3500.00 1114.80
8.90 3000.00 1005.90
8.40 2500.00  890.80
7.80 2000.00  767.70
7.10 1500.00  633.70
6.20 1000.00  483.60

6. A 70 lb bag of rice is being pulled by a person by applying a force $F$ at an angle $\theta$ as shown. The force required to drag the bag is given by:

$$F(\theta) = \frac{70\mu}{\mu \sin \theta \cos \theta}$$

where $\mu = 0.35$ is the friction coefficient.

(a) Determine $F(\theta)$ for $\theta = 5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ$.
(b) Determine the angle $\theta$ where $F$ is minimum. Do it by creating a vector $\theta$ with elements ranging from $5^\circ$ to $35^\circ$ and spacing of 0.01. Calculate $F$ for each value of $\theta$ and then find the maximum $F$ and associated $\theta$ with MATLAB's built-in function $\text{max}$.

Solution

Script File:
th=5:5:35; mu=0.35;
% Part (a)
F=70*mu.(/(mu*sind(th)+cosd(th))
% Part (b)
thb=5:0.01:35;
Fb=70*mu.(/(mu*sind(thb)+cosd(thb)));
[
[Fm, i] = min(Fb);
Fmin = Fb(i)
at_Theta = thb(i)

Command Window:

F =
23.86    23.43    23.19    23.13
23.24    23.53    24.02
Fmin =
23.12
at_Theta =
19.29

7. The remaining loan balance, $B$, of a fixed payment $n$ years mortgage after $x$ years is given by:

$$B = \frac{L\left((1 + \frac{r}{12})^{2n} - ((1 + \frac{r}{12})^{2x})\right)}{((1 + \frac{r}{12})^{2n} - 1)}$$

where $L$ is the loan amount, and $r$ is the annual interest rate. Calculate the balance of a 30 year, $100,000$ mortgage, with annual interest rate of 6% (use 0.06 in the equation) after 0, 5, 10, 15, 20, 25, and 30 years. Create a seven-element vector for $x$ and use element-by-element operations. Display the results in a two-row table where the values of years, and balance are displayed in the first and second rows, respectively.

Solution

Script File:

L=100000; n=30; r=0.06;
x=0:5:30;
C=(1+r/12);
B=L*(C^(12*n)-C.(12*x))./(C^(12*n)-1);
Table=[x;B]

Command Window:
Table =
0                 5.00         10.00
15.00    20.00         25.00         30.00
100000.00    93054.36    83685.72    71048.84
54003.59    31012.09           0
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8. The length $|\mathbf{u}|$ (magnitude) of a vector $\mathbf{u} = xi + yj + zk$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = -5.6\mathbf{i} + 11\mathbf{j} - 14\mathbf{k}$, determine its length by writing one MATLAB command in which the vector is multiplied by itself using element-by-element operation and the MATLAB built-in functions \texttt{sum} and \texttt{sqrt} are used.

Solution

$$\mathbf{u} = [-5.6 \ 11 \ -14]$$

$$\text{Length} = \sqrt{\text{sum}(\mathbf{u}.*\mathbf{u})}$$

$$\text{Length} = 18.6644$$

9. A unit vector $\mathbf{u}_n$ in the direction of the vector $\mathbf{u} = xi + yj + zk$ is given by $\mathbf{u}_n = \mathbf{u} / |\mathbf{u}|$ where $|\mathbf{u}|$ is the length (magnitude) of the vector, given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = 4\mathbf{i} + 13\mathbf{j} - 7\mathbf{k}$, determine the unit vector in the direction of $\mathbf{u}$ using the following steps:

(a) Assign the vector to a variable $\mathbf{u}$.

(b) Using element-by-element operation and the MATLAB built-in functions \texttt{sum} and \texttt{sqrt} calculate the length of $\mathbf{u}$ and assign it to the variable $Lu$.

(c) Use the variables from Parts (a) and (b) to calculate $\mathbf{u}_n$.

(d) Verify that the length of $\mathbf{u}_n$ is 1 (same operations as in Part (b)).

Solution

$$\mathbf{u} = [4 \ 13 \ -7]$$

$$Lu = \sqrt{\text{sum}(\mathbf{u}.*\mathbf{u})}$$

$$Lu = 15.2971$$

$$\mathbf{u}_n = \mathbf{u} / Lu$$

$$\mathbf{u}_n = \begin{bmatrix} 0.2615 \\ 0.8498 \\ -0.4576 \end{bmatrix}$$

$$Lun = \sqrt{\text{sum}(\mathbf{u}_n.*\mathbf{u}_n)}$$

$$Lun = 1.0000$$
10. The angle between two vectors \( \mathbf{u}_1 = x_1 \mathbf{i} + y_1 \mathbf{j} + z_1 \mathbf{k} \) and \( \mathbf{u}_2 = x_2 \mathbf{i} + y_2 \mathbf{j} + z_2 \mathbf{k} \) can be determined by 
\[
\cos \theta = \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{|\mathbf{u}_1||\mathbf{u}_2|},
\]
where \( |\mathbf{u}_1| = \sqrt{x_1^2 + y_1^2 + z_1^2} \).

Given the vectors \( \mathbf{u}_1 = 3.2 \mathbf{i} - 6.8 \mathbf{j} + 9 \mathbf{k} \) and \( \mathbf{u}_2 = -4 \mathbf{i} + 2 \mathbf{j} + 7 \mathbf{k} \), determine the angle between them (in degrees) by writing one MATLAB command that uses element-by-element multiplication and the MATLAB built-in functions \texttt{acosd}, \texttt{sum} and \texttt{sqrt}.

**Solution**

```matlab
>> u1=[3.2 -6.8 9];
>> u2=[-4 2 7];
>> Theta=acosd(sum(u1.*u2)/(sqrt(sum(u1.*u1))*sqrt(sum(u2.*u2))))
```

\[ \Theta = 67.93 \]

11. The following vector is defined in MATLAB:
\[ \mathbf{d} = [2 \; 4 \; 3] \]

By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB. Check your answers by executing the commands with MATLAB.

\( (a) \) \( \mathbf{d} + \mathbf{d} \) \hspace{1cm} \( (b) \) \( \mathbf{d} \cdot \mathbf{d} \) \hspace{1cm} \( (c) \) \( \mathbf{d} \cdot \mathbf{d} \) \hspace{1cm} \( (d) \) \( \mathbf{d} \cdot \mathbf{d}^2 \)

**Solution**

```matlab
>> d=[2 4 3];
>> d+d
ans =
   4.00   8.00   6.00
>> d.^d
ans =
   4.00  256.00  27.00
>> d.*d
ans =
   4.00  16.00  9.00
>> d.^2
ans =
   4.00  16.00  9.00
```
12. The following two vectors are defined in MATLAB:
\[ v = [3 \quad -1 \quad 2], \quad u = [6 \quad 4 \quad -3] \]
By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB. Check your answers by executing the commands with MATLAB.
(a) \( v \cdot u \)
(b) \( v^\cdot u \)
(c) \( v \cdot u' \)

Solution

```matlab
>> v=[3 -1 2];
>> u=[6 4 -3];
>> v.*u
ans =
    18   -4   -6
>> v.^u
ans =
   729.0000    1.0000    0.1250
>> v*u'
ans =
    8
```

13. Define the vector \( v = [1 \quad 3 \quad 5 \quad 7] \). Then use the vector in a mathematical expression to create the following vectors:
(a) \( a = [3 \quad 9 \quad 15 \quad 21] \)
(b) \( b = [1 \quad 9 \quad 25 \quad 49] \)
(c) \( c = [1 \quad 1 \quad 1 \quad 1] \)
(d) \( d = [6 \quad 6 \quad 6 \quad 6] \)

Solution

```matlab
>> v=[1 3 5 7];
>> a=3*v
a =
    3     9    15    21
>> b=v.^2
b =
    1     9    25    49
>> c=v./v
C =
    1     1     1     1
>> d=6*v./v
D =
    6     6     6     6
```
14. Define the vector \( v = [5 \quad 4 \quad 3 \quad 2] \). Then use the vector in a mathematical expression to create the following vectors:

\[
\begin{align*}
(a) \quad a & = \left[ \frac{1}{5+5} \quad \frac{1}{4+4} \quad \frac{1}{3+3} \quad \frac{1}{2+2} \right] \\
(b) \quad b & = [5^4 \quad 4^3 \quad 3^2 \quad 2^1] \\
(c) \quad c & = \left[ \frac{5}{\sqrt{5}} \quad \frac{4}{\sqrt{4}} \quad \frac{3}{\sqrt{3}} \quad \frac{2}{\sqrt{2}} \right] \\
(d) \quad d & = \left[ \frac{5^2}{5^4} \quad \frac{4^2}{4^4} \quad \frac{3^2}{3^4} \quad \frac{2^2}{2^4} \right]
\end{align*}
\]

Solution

\[
\begin{align*}
\text{>> } v &= [5 \quad 4 \quad 3 \quad 2] \\
\text{>> } a &= 1./(v+v) \\
a &= \begin{bmatrix} 0.1000 & 0.1250 & 0.1667 & 0.2500 \end{bmatrix} \\
\text{>> } b &= v.^v \\
b &= \begin{bmatrix} 3125 & 256 & 27 & 4 \end{bmatrix} \\
\text{>> } c &= v./sqrt(v) \\
c &= \begin{bmatrix} 2.2361 & 2.0000 & 1.7321 & 1.4142 \end{bmatrix} \\
\text{>> } d &= v.^2./v.^v \\
d &= \begin{bmatrix} 0.0080 & 0.0625 & 0.3333 & 1.0000 \end{bmatrix}
\end{align*}
\]

15. Define \( x \) and \( y \) as the vectors \( x = [0.5, 1, 1.5, 2, 2.5] \) and \( y = [0.8, 1.6, 2.4, 3.2, 4.0] \). Then use them in the following expressions to calculate \( z \) using element-by-element calculations.

\[
\begin{align*}
(a) \quad z & = x^2 + 2xy \\
(b) \quad z & = xy^{y/x} - \sqrt[3]{x^4 y^3 + 8.5}^{1/3}
\end{align*}
\]

Solution

\[
\begin{align*}
\text{>> } x &= 0.5:0.5:2.5 \\
\text{>> } y &= 0.8:0.8:4; \\
\text{>> } % \text{ Part (a):} \\
\text{>> } z &= x.^2+2*x.*y \\
z &= \begin{bmatrix} 1.0500 & 4.2000 & 9.4500 & 16.8000 & 26.2500 \end{bmatrix} \\
\text{>> } % \text{ Part (b):} \\
\text{>> } z &= x.*y.*exp(y./x) - (x.^4.*y.^3+8.5).^(1/3) \\
z &= \begin{bmatrix} -0.0622 & 5.5981 & 13.5494 & 23.5926 & 35.9429 \end{bmatrix}
\end{align*}
\]
16. Define \( r \) and \( s \) as scalars \( r = 1.6 \times 10^3 \) and \( s = 14.2 \), and \( t, x, \) and \( y \) as vectors \( t = [1, 2, 3, 4, 5] \), \( x = [2, 4, 6, 8, 10] \), and \( y = [3, 6, 9, 12, 15] \). Then use these variables to calculate the following expressions using element-by-element calculations for the vectors.

\[
(a) \quad G = xt + \frac{r}{s} (y^2 - x)t \\
(b) \quad R = \frac{r(-xt + y^2)}{15} - s^2 (y - 0.5x^2)t
\]

Solution

\[
\begin{align*}
&\text{>> } r = 1.6 \times 10^3; \quad s = 14.2; \\
&\text{>> } t = 1:5; \quad \text{>> } x = 2:2:10; \quad \text{>> } y = 3:3:15; \\
&\text{>> } \% \text{ Part (a):} \\
&\text{>> } G = x \cdot t + r/s^2 \cdot (y^2 - x) \cdot t \\
&\quad G = \\
&\quad 1.0e+03 \times \\
&\quad 0.0575 \quad 0.5158 \quad 1.8034 \quad 4.3486 \quad 8.5801 \\
&\text{>> } \% \text{ Part (b):} \\
&\text{>> } R = r \times (-x \cdot t + y \cdot t \cdot .^2) / 15 - s^2 \times (y - 0.5 \times x \cdot .^2) \cdot t \\
&\quad R = \\
&\quad 1.0e+04 \times \\
&\quad -0.0095 \quad 0.2513 \quad 1.2164 \quad 3.3198 \quad 6.9954
\end{align*}
\]

17. The area of a triangle \( ABC \) can be calculated by \( |r_{AB} \times r_{AC}| / 2 \), where \( r_{AB} \) and \( r_{AC} \) are vectors connecting the vertices \( A \) and \( B \), and \( A \) and \( C \), respectively. Determine the area of the triangle shown in the figure. Use the following steps in a script file to calculate the area. First, define the vectors \( r_{OA}, r_{OB} \) and \( r_{OC} \) from knowing the coordinates of points \( A, B, \) and \( C \). Then determine the vectors \( r_{AB} \) and \( r_{AC} \) from \( r_{OA}, r_{OB} \) and \( r_{OC} \). Finally, determine the area by using MATLAB’s built-in functions \texttt{cross}, \texttt{sum}, and \texttt{sqrt}.

Solution

\[
\begin{align*}
&\text{>> } rOA = [8 \ 5 \ -4]; \\
&\text{>> } rOB = [-7 \ 9 \ 6]; \\
&\text{>> } rOC = [-5 \ -2 \ 11]; \\
&\text{>> } rAB = rOB - rOA;
\end{align*}
\]
18. The cross product of two vectors can be used for determining the angle between two vectors:

\[ \theta = \sin^{-1}\left( \frac{|\mathbf{r}_1 \times \mathbf{r}_2|}{|\mathbf{r}_1||\mathbf{r}_2|} \right) \]

Use MATLAB’s built-in functions `asind`, `cross`, `sqrt`, and `dot` to find the angle (in degrees) between \( \mathbf{r}_1 = 2.5\mathbf{i} + 8\mathbf{j} - 5\mathbf{k} \) and \( \mathbf{r}_2 = -\mathbf{i} + 6\mathbf{j} + 3\mathbf{k} \). Recall that \( |\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}} \).

Solution

```
>> r1=[2.5 8 -5];
>> r2=[-1 6 3];
>> r1CRr2=cross(r1,r2);
>> r1CRr2L=sqrt(sum(r1CRr2.*r1CRr2));
>> r1L=sqrt(sum(r1.*r1));
>> r2L=sqrt(sum(r2.*r2));
>> Theta=asind(r1CRr2L/(r1L*r2L))
Theta =
   62.5629
```

19. The center of mass, \( (\bar{x}, \bar{y}, \bar{z}) \), of \( n \) particles can be calculated by:

\[
\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}, \quad \bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}, \quad \bar{z} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i}
\]

where \( x_i, y_i, \) and \( z_i \) and \( m_i \) are the coordinates and the mass of particle \( i \), respectively. The coordinates and mass of six particles are listed in the following table. Calculate the center of mass of the particles.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (kg)</th>
<th>Coordinate x (mm)</th>
<th>Coordinate y (mm)</th>
<th>Coordinate z (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>-10</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>B</td>
<td>0.8</td>
<td>-18</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>-7</td>
<td>11</td>
<td>2</td>
</tr>
</tbody>
</table>
Solution

Script File:
% P is a matrix with 6 rows and 4 columns.
% A row for each particle.
% Columns: mass, x, y, z coordinates
P=[0.5 -10 8 32; 0.8 -18 6 19; 0.2 -7 11 2
   1.1 5 12 -9; 0.4 0 -8 -6; 0.9 25 -20 8];
Mass=sum(P(:,1))
x=sum(P(:,1).*P(:,2))/Mass
y=sum(P(:,1).*P(:,3))/Mass
z=sum(P(:,1).*P(:,4))/Mass

Command Window:

Mass =
   3.9000
x =
   1.8462
y =
   0.7692
z =
   6.7949

20. Define the vectors:
   \[ \mathbf{a} = 7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}, \quad \mathbf{b} = -4\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}, \text{ and } \mathbf{c} = 5\mathbf{i} - 6\mathbf{j} + 8\mathbf{k} \]
Use the vectors to verify the identity:
   \[ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b} (\mathbf{a} \cdot \mathbf{c}) - \mathbf{c} (\mathbf{a} \cdot \mathbf{b}) \]
Use MATLAB's built-in functions cross and dot, calculate the value of the left and right sides of the identity.

Solution

>> a=[7 -4 6];
>> b=[-4 7 5];
>> c=[5 -6 8];
>> LHS=cross(a,cross(b,c))
LHS =
\[-298 \quad 593 \quad 743\]
>> RHS=b*dot(a,c)-c*dot(a,b)
RHS =
\[-298 \quad 593 \quad 743\]

21. The maximum distance $s$ and the maximum height $h$ that a projectile shot at an angle $\theta$ are given by:

$$ s = \frac{v_0^2 \sin 2\theta}{g} \quad \text{and} \quad h = \frac{v_0^2 \sin^2 \theta}{2g} $$

where $v_0$ is the shooting velocity and $g = 9.81 \text{ m/s}^2$. Determine $s(\theta)$ for $\theta = 15^\circ, 25^\circ, 35^\circ, 45^\circ, 55^\circ, 65^\circ, 75^\circ$ if $v_0 = 260\text{ m/s}$.

**Solution**

```
>> g=9.81; v0=260;
>> th=15:10:75
th =
15   25   35   45   55   65   75
>> s=v0^2/g*sind(2*th)
s =
1.0e+03 *
   3.4455   5.2788   6.4754   6.8909   6.4754
   5.2788   3.4455
>> h=v0^2*sind(th).^2/(2*g)
h =
1.0e+03 *
   0.2308   0.6154   1.1335   1.7227   2.3119
   2.8301   3.2147
```
22. Use MATLAB to show that the sum of the infinite series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges to \( \pi^2 / 6 \). Do this by computing the sum for:

(a) \( n = 5 \),  
(b) \( n = 50 \),  
(c) \( n = 5000 \)
For each part create a vector \( n \) in which the first element is 1, the increment is 1 and the last term is 5, 50, or 5000. Then use element-by-element calculations to create a vector in which the elements are \( \frac{1}{n^2} \). Finally, use MATLAB’s built-in function `sum` to sum the series. Compare the values to \( \pi^2 / 6 \).

Use `format long` to display the numbers.

**Solution**

Script File:

```matlab
format long
% Part (a)
n=1:5;
an=1./n.^2;
Sa=sum(an)
pi_sq_ov6=pi^2/6
% Part (b)
n=1:50;
an=1./n.^2;
Sb=sum(an)
pi_sq_ov6=pi^2/6
% Part (c)
n=1:5000;
an=1./n.^2;
Sc=sum(an)
pi_sq_ov6=pi^2/6
```

Command Window:

```
Sa =
  1.463611111111111
pi_sq_ov6 =
  1.644934066848226
Sb =
  1.62513273621529
pi_sq_ov6 =
```
1.644934066848226
Sc =
1.644734086846901
pi_sq_ov6 =
1.644934066848226

23. Use MATLAB to show that the sum of the infinite series \( \sum_{n=1}^{\infty} \frac{n^2}{2^n} \) converges to 6. Do this by computing the sum for
(a) \( n = 5 \), (b) \( n = 15 \), (c) \( n = 30 \)
For each part, create a vector \( n \) in which the first element is 1, the increment is 1 and the last term is 5, 15 or 30. Then use element-by-element calculations to create a vector in which the elements are \( \frac{n^2}{2^n} \). Finally, use MATLAB's built-in function sum to sum the series. Use \texttt{format long} to display the numbers.

Solution

Script File:
\begin{verbatim}
format long
% Part (a)
n=1:5;
an=n.^2./2.^n;
Sa=sum(an)
% Part (b)
n=1:15;
an=n.^2./2.^n;
Sb=sum(an)
% Part (c)
n=1:30;
an=n.^2./2.^n;
Sc=sum(an)
\end{verbatim}

Command Window:
\begin{verbatim}
Sa =
 4.406250000000000
Sb =
 5.991119384765625
Sc =
 5.99999904463038
\end{verbatim}
24. The natural exponential function can be expressed by \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \). Determine \( e^2 \) by calculating the sum of the series for:

- (a) \( n = 5 \)
- (b) \( n = 15 \)
- (c) \( n = 25 \)

For each part create a vector \( n \) in which the first element is 0, the increment is 1, and the last term is 5, 15, or 25. Then use element-by-element calculations to create a vector in which the elements are \( \frac{x^n}{n!} \). Finally, use the MATLAB built-in function \( \text{sum} \) to add the terms of the series. Compare the values obtained in parts (a), (b), and (c) with the value of \( e^2 \) calculated by MATLAB.

**Solution**

Script File:

```matlab
format long
% Part (a)
 n=0:5;
an=2.\^n./factorial(n);
Sa=sum(an)
ePower2=exp(2)
% Part (b)
 n=0:15;
an=2.\^n./factorial(n);
Sb=sum(an)
ePower2=exp(2)
% Part (c)
 n=0:25;
an=2.\^n./factorial(n);
Sc=sum(an)
ePower2=exp(2)
```

Command Window:

```
Sa = 7.266666666666667
ePower2 = 7.389056098930650
Sb = 7.389056095384136
ePower2 =
```
Chapter 3: Solved Problems

25. Show that \( \lim_{x \to \pi/3} \frac{\sin(x - \pi/3)}{4 \cos^2 x - 1} = \frac{-\sqrt{3}}{6} \). Do this by first creating a vector \( x \) that has the elements \( \pi/3 - 0.1, \pi/3 - 0.01, \pi/3 - 0.001, \pi/3 + 0.001, \pi/3 + 0.01, \) and \( \pi/3 + 0.1 \). Then, create a new vector \( y \) in which each element is determined from the elements of \( x \) by \( \frac{\sin(x - \pi/3)}{4 \cos^2 x - 1} \). Compare the elements of \( y \) with the value \( \frac{-\sqrt{3}}{6} \). Use format long to display the numbers.

Solution

Script File:

```matlab
format long
x=[pi/3-0.1 pi/3-0.01 pi/3-0.0001 pi/3+0.0001 pi/3+0.01 pi/3+0.1];
Lim=sin(x-pi/3)./(4*cos(x).^2-1)
CompareValue=sqrt(3)/6
```

Command Window:

```
Lim =
Columns 1 through 4
-0.274238398404589 -0.287032331771222 -
0.288658470333254 -0.288691803667625
Columns 5 through 6
-0.290366054030767 -0.307964386863987
CompareValue =
0.288675134594813
```

26. Show that \( \lim_{x \to 0} \frac{5 \sin(6x)}{4x} = 7.5 \). Do this by first creating a vector \( x \) that has the elements 1.0, 0.1, 0.01, 0.001, and 0.0001. Then, create a new vector \( y \) in which each element is determined from the elements of \( x \) by \( \frac{5 \sin(6x)}{4x} \).
Chapter 3: Solved Problems

Compare the elements of \( y \) with the value 7.5. Use format long to display the numbers.

Solution

Command Window:

\[
\begin{align*}
&>> x = [1 \ 0.1 \ 0.01 \ 0.001 \ 0.0001]; \\
&>> y = 5 \times \sin(6 \times x) / (4 \times x) \\
& y = \\
& \begin{array}{lllllll}
& \text{Columns 1 through 4} \\
& -0.349269372748657 & 7.058030917437943 \\
& 7.495500809930575 & 7.499955000080999 \\
& \text{Column 5} \\
& 7.499999550000009
\end{array}
\end{align*}
\]

27. The Hazen Williams equation can be used to calculate the pressure drop, \( P_d \) (psi/ft of pipe) in pipes due to friction:

\[
P_d = \frac{4.52Q^{1.85}}{(C^{1.85}d^{4.87})}
\]

where \( Q \) is the flow rate (gpm), \( C \) is a design coefficient determined by the type of pipe, and \( d \) is pipe diameter in inches. Consider a 3.5 in. diameter steel pipe with \( C = 120 \). Calculate the pressure drop in a 1000 ft long pipe for flow rates of 250, 300, 350, 400, and 450 gpm. To carry out the calculation first create a five-element vector with the values of the flow rates (250, 300, ...). Then use the vector in the formula using element-by-element operations.

Solution

\[
\begin{align*}
&>> C=120; d=3.5; \\
&>> Q=250:50:450 \\
& Q = \\
& \begin{array}{llllll}
& 250 \ 300 \ 350 \ 400 \ 450
\end{array} \\
& >> Pd=4.52*Q.^{1.85}/(C^{1.85}*d^{4.87}) \\
& Pd = \\
& \begin{array}{lllllll}
& \text{Columns 1 through 4} \\
& 0.039375925047993 & 0.055171663010881 \\
& 0.073378298262268 & 0.093940471923615 \\
& \text{Column 5} \\
& 0.116811312607597
\end{array}
\end{align*}
\]
28. The monthly lease payment, \( Pmt \), of a new car can be calculated by:

\[
Pmt = \frac{Pv - \frac{Fv}{(1+\frac{i}{12})^N}}{\frac{1}{1+\frac{i}{12})^N}}
\]

where \( Pv \) and \( Fv \) are the present value and the future value (at the end of the lease) of the car, respectively. \( N \) is the duration of the lease in months, and \( i \) is the interest rate per year. Consider a 36 months lease of a car with a present value of $38,000 and a future value of $23,400. Calculate the monthly payments if the yearly interest rates are 3, 4, 5, 6, 7, and 8\%. To carry out the calculation first create a five-element vector with the values of the interest rates (0.03, 0.04, ....). Then use the vector in the formula using element-by-element operations.

**Solution**

\[
>> i=0.03:0.01:0.08;
>> Nu=38000-23400./(1+i/12).^36;
>> De=(1-1./(1+(1+i./12).^36))./(i/12);
>> Pmt=Nu./De
\]

\[\text{Pmt} =
\begin{array}{cccc}
0.794879505046213 & 1.084572758996790 & 1.384345790655600 \\
1.693006947249476 & 2.009471591203944 & 2.332753536801580
\end{array}
\]
Chapter 3: Solved Problems

29. Create the following three matrices:

\[
A = \begin{bmatrix}
5 & -3 & 7 \\
1 & 0 & -6 \\
-4 & 8 & 9
\end{bmatrix}
B = \begin{bmatrix}
3 & 2 & -1 \\
6 & 8 & -7 \\
4 & 4 & 0
\end{bmatrix}
C = \begin{bmatrix}
-9 & 8 & 3 \\
1 & 7 & -5 \\
3 & 3 & 6
\end{bmatrix}
\]

(a) Calculate \( A + B \) and \( B + A \) to show that addition of matrices is commutative.

(b) Calculate \( A(B*C) \) and \( (A*B)*C \) to show that multiplication of matrices is associative.

(c) Calculate \( 5(B+C) \) and \( 5B+5C \) to show that, when matrices are multiplied by a scalar, the multiplication is distributive.

(d) Calculate \( (A+B)*C \) and \( A*C+B*C \) to show that matrix multiplication is distributive.

Solution

Script File:

\[
A = \begin{bmatrix}
5 & -3 & 7 \\
1 & 0 & -6 \\
-4 & 8 & 9
\end{bmatrix}
B = \begin{bmatrix}
3 & 2 & -1 \\
6 & 8 & -7 \\
4 & 4 & 0
\end{bmatrix}
C = \begin{bmatrix}
-9 & 8 & 3 \\
1 & 7 & -5 \\
3 & 3 & 6
\end{bmatrix}
\]

\[
ApB = A + B
BpA = B + A
AtBC = A(B*C)
ABtC = (A*B)*C
BpC5 = 5*(B+C)
B5pC5 = 5*B+5*C
ApBtC = (A+B)*C
AtCpBtC = A*C+B*C
\]

Command Window:

\[
A = \\
\begin{bmatrix}
5 & -3 & 7 \\
1 & 0 & -6 \\
-4 & 8 & 9
\end{bmatrix}
B = \\
\begin{bmatrix}
3 & 2 & -1 \\
6 & 8 & -7 \\
4 & 4 & 0
\end{bmatrix}
C = \\
\begin{bmatrix}
-9 & 8 & 3
\end{bmatrix}
\]
\[
\begin{array}{ccc}
1 & 7 & -5 \\
3 & 3 & 6 \\
\end{array}
\]

\[
\text{ApB} = \\
\begin{array}{ccc}
8 & -1 & 6 \\
7 & 8 & -13 \\
0 & 12 & 9 \\
\end{array}
\]

\[
\text{BpA} = \\
\begin{array}{ccc}
8 & -1 & 6 \\
7 & 8 & -13 \\
0 & 12 & 9 \\
\end{array}
\]

\[
\text{AtBC} = \\
\begin{array}{ccc}
-163 & 346 & 101 \\
164 & -325 & 41 \\
-712 & 1064 & -556 \\
\end{array}
\]

\[
\text{ABtC} = \\
\begin{array}{ccc}
-163 & 346 & 101 \\
164 & -325 & 41 \\
-712 & 1064 & -556 \\
\end{array}
\]

\[
\text{BpC5} = \\
\begin{array}{ccc}
-30 & 50 & 10 \\
35 & 75 & -60 \\
35 & 35 & 30 \\
\end{array}
\]

\[
\text{BSpC5} = \\
\begin{array}{ccc}
-30 & 50 & 10 \\
35 & 75 & -60 \\
35 & 35 & 30 \\
\end{array}
\]

\[
\text{ApBtC} = \\
\begin{array}{ccc}
-55 & 75 & 65 \\
-94 & 73 & -97 \\
39 & 111 & -6 \\
\end{array}
\]

\[
\text{AtCpBtC} = \\
\begin{array}{ccc}
-55 & 75 & 65 \\
-94 & 73 & -97 \\
39 & 111 & -6 \\
\end{array}
\]
30. Use the matrices $A$, $B$, and $C$ from the previous problem to answer the following:

(a) Does $A^*B = B^*A$?  
(b) Does $(B^*C)^{-1} = B^{-1}C^{-1}$?  
(c) Does $(A^{-1})' = (A')^{-1}$? (‘ means transpose)  
(d) Does $(A + B)^t = A^t + B^t$?

Solution

Script File:

```matlab
A=[5 -3 7; 1 0 -6; -4 8 9]
B=[3 2 -1; 6 8 -7; 4 4 0]
C=[-9 8 3; 1 7 -5; 3 3 6]
AtB=A*B
BtA=B*A
BtCi=(B*C)^-1
BitCi=B^-1*C^-1
Ait=A^-1'
Ati=A'^-1
ApBt=(A+B)'
AtpBt=A'+B'
```

Command Window:

```
A =
  5    -3     7
  1     0    -6
 -4     8     9
B =
  3     2    -1
  6     8    -7
  4     4     0
C =
 -9     8     3
  1     7    -5
  3     3     6
AtB =
  25    14    16
 -21   -22    -1
  72    92   -52
BtA =
  21   -17     0
  66   -74   -69
  24   -12     4
BtCi =
 -0.1200    0.0053    0.0627
```
31. Create a $3 \times 3$ matrix $A$ having random integer values between 1 and 5. Call the matrix $A$ and, using MATLAB, perform the following operations. For each part explain the operation.

(a) $A.^A$
(b) $A.*A$
(c) $A*A-1$
(d) $A./A$
(e) $\text{det}(A)$
(f) $\text{inv}(A)$

**Solution**

```
>> A=randi(5,3)
A =
    1     3     4
    5     1     2
    3     5     3
>> A.^A
ans =
    1     27    256
      3125     1     4
     27   3125     27
>> A.*A
ans =
```
32. The magic square is an arrangement of numbers in a square grid in such a way that the sum of the numbers in each row, and in each column, and in each diagonal is the same. MATLAB has a built-in function `magic(n)` that returns an $n \times n$ magic square. In a script file create a $(5 \times 5)$ magic square, and then test the properties of the resulting matrix by finding the sum of the elements in each row, in each column and in both diagonals. In each case, use MATLAB’s built-in function `sum`. (Other functions that can be useful are `diag` and `fliplr`.)

Solution

Script File:

```matlab
A = magic(5)
AR1 = sum(A(1,:))
AR2 = sum(A(2,:))
AR3 = sum(A(3,:))
AR4 = sum(A(4,:))
AR5 = sum(A(5,:))
AC1 = sum(A(:,1))
AC2 = sum(A(:,2))
```
AC3 = sum(A(:,3))
AC4 = sum(A(:,4))
AC5 = sum(A(:,5))
DAR = sum(diag(A))
DAL = sum(diag(fliplr(A)))

Command Window:
A =
    17  24   1   8  15
    23   5   7  14  16
     4   6  13  20  22
    10  12  19  21   3
    11  18  25   2   9
AR1 =
     65
AR2 =
     65
AR3 =
     65
AR4 =
     65
AR5 =
     65
AC1 =
     65
AC2 =
     65
AC3 =
     65
AC4 =
     65
AC5 =
     65
DAR =
     65
DAL =
     65

33. Solve the following system of three linear equations:

\[-2x + 5y + 7z = -17.5
\] 
\[3x - 6y + 2z = 40.6
\] 
\[9x - 3y + 8z = 56.2
\]

Solution

```matlab
>> a = [-2 5 7; 3 -6 2; 9 -3 8];
```
34. Solve the following system of six linear equations:

\[
\begin{align*}
2a - 4b + 5c - 3.5d + 1.8e + 4f &= 52.52 \\
-1.5a + 3b - 4c - d - 2e + 5f &= -21.1 \\
5a + b - 6c + 3d - 2e + 2f &= -27.6 \\
1.2a - 2b + 3c + 4d - e + 4f &= 9.16 \\
4a + b - 2c - 3d - 4e + 1.5f &= -17.9 \\
3a + b - c + 4d - 2e - 4f &= -16.2
\end{align*}
\]

Solution

\[
\begin{align*}
>> & \quad \text{a} = \begin{bmatrix} 2 & -4 & 5 & -3.5 & 1.8 & 4; -1.5 & 3 & 4 & -1 & -2 & 5; 5 & 1 & -6 & 3 & -2 & 2 \\
& & 1.2 & -2 & 3 & 4 & -1 & 4; 4 & 1 & -2 & -3 & -4 & 1.5; 3 & 1 & -1 & 4 & -2 & -4 \end{bmatrix} \\
& \quad \text{a} = \\
& \quad \begin{bmatrix} 2.0000 & -4.0000 & 5.0000 & -3.5000 & 1.8000 \\
& 4.0000 & -1.5000 & 3.0000 & 4.0000 & -1.0000 & -2.0000 \\
& 5.0000 & 5.0000 & 1.0000 & -6.0000 & 3.0000 & -2.0000 \\
& 2.0000 & 1.2000 & -2.0000 & 3.0000 & 4.0000 & -1.0000 \\
& 4.0000 & 4.0000 & 1.0000 & -2.0000 & -3.0000 & -4.0000 \\
& 1.5000 & 3.0000 & 1.0000 & -1.0000 & 4.0000 & -2.0000 - \\
& 4.0000 \end{bmatrix} \\
>> & \quad \text{b} = \begin{bmatrix} 52.52; -21.1; -27.6; 9.16; -17.9; -16.2 \end{bmatrix} \\
b = \\
& \quad \begin{bmatrix} 52.5200 \\
& -21.1000 \\
& -27.6000 \\
& 9.1600 \\
& -17.9000 \\
& -16.2000 \end{bmatrix} \\
>> & \quad \text{x} = \text{a} \backslash \text{b} \\
x = \\
& \quad \begin{bmatrix} 1.8000 \end{bmatrix}
\end{align*}
\]
35. A football stadium has 100,000 seats. In a game with full capacity people with the following ticket and associated cost attended the game:

<table>
<thead>
<tr>
<th>Cost</th>
<th>Student</th>
<th>Alumni</th>
<th>Faculty</th>
<th>Public</th>
<th>Veterans</th>
<th>Guests</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25</td>
<td>$40</td>
<td>$60</td>
<td>$70</td>
<td>$32</td>
<td>$0</td>
<td></td>
</tr>
</tbody>
</table>

Determine the number of people that attended the game in each cost category if the total revenue was $4,897,000, there were 11,000 more alumni than faculty, the number of public plus alumni together was 10 times the number of veterans, the number of faculty plus alumni together was the same as the number of students, and the number of faculty plus students together was four times larger than the number of guests and veterans together.

Solution

\[ S + A + F + P + V + G = 100000 \]
\[ 25S + 40A + 60F + 70P + 32V + 0G = 4897000 \]
\[ F + 11000 = A \]
\[ P + A = 10V \]
\[ F + A = S \]
\[ F + S = 4(V + G) \]

```matlab
>> a=[1 1 1 1 1 1; 25 40 60 70 32 0; 0 -1 1 0 0 0; 0 1 0 1 -10 0 -1 1 0 0 0; 1 0 1 0 -4 -4]
>> b=[100000; 4897000; -11000; 0; 0; 0]
```

```matlab
a =
    1     1     1     1     1     1
    25    40    60    70    32     0
    0    -1     1     0     0     0
    0     1     0     1   -10     0
   -1     1     1     0     0     0
    1     0     1     0   -4    -4
>> b=[100000; 4897000; -11000; 0; 0; 0]
```

```matlab
b =
   100000
   4897000
  -11000
     0
     0
```
36. A food company manufactures five types of 8 oz Trail mix packages using different mixtures of peanuts, almonds, walnuts, raisins, and M&Ms. The mixtures have the following compositions:

<table>
<thead>
<tr>
<th>Mix</th>
<th>Peanuts (oz)</th>
<th>Almonds (oz)</th>
<th>Walnuts (oz)</th>
<th>Raisins (oz)</th>
<th>M&amp;Ms (oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mix 1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Mix 2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Mix 3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Mix 4</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Mix 5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

How many packages of each mix can be manufactured if 105 lb of peanuts, 74 lb of almonds, 102 lb of walnuts, 118 lb of raisins, and 121 lb of M&Ms are available? Write a system of linear equations and solve.

**Solution**

\[3a + b + c + 2d + e = 105 * 16\]
\[a + 2b + c + 0d + 2e = 74 * 16\]
\[a + b + 0c + 3d + 3e = 102 * 16\]
\[2a + 3b + 3c + d + 0e = 118 * 16\]
\[a + b + 3c + 2d + 2e = 121 * 16\]

\[
\begin{bmatrix}
3 & 1 & 1 & 2 & 1 \\
1 & 2 & 1 & 0 & 2 \\
1 & 1 & 0 & 3 & 3 \\
2 & 3 & 3 & 1 & 0 \\
1 & 1 & 3 & 2 & 2
\end{bmatrix}
\]

\[
a =
\begin{bmatrix}
3 & 1 & 1 & 2 & 1 \\
1 & 2 & 1 & 0 & 2 \\
1 & 1 & 0 & 3 & 3 \\
2 & 3 & 3 & 1 & 0
\end{bmatrix}
\]
1     1     3     2     2
>> b=[105; 74; 102; 118; 121]*16
b =
   1680
   1184
   1632
   1888
   1936
>> x=a\b
x =
   208.0000
   176.0000
   240.0000
   224.0000
   192.0000


37. The electrical circuit shown consists of resistors and voltage sources. Determine \( i_1, i_2, i_3 \) and \( i_4 \), using the mesh current method based on Kirchhoff’s voltage law (see Sample Problem 3-4).

\[ V_1 = 28 \, \text{V}, \quad V_2 = 36 \, \text{V}, \quad V_3 = 42 \, \text{V}, \]
\[ R_1 = 16 \, \Omega, \quad R_2 = 10 \, \Omega, \quad R_3 = 6 \, \Omega, \]
\[ R_4 = 12 \, \Omega, \quad R_5 = 8 \, \Omega, \quad R_6 = 14 \, \Omega, \]
\[ R_7 = 4 \, \Omega, \quad R_8 = 5 \, \Omega. \]

Solution

\[
\begin{align*}
V_1 - R_1 i_1 - R_3 (i_1 - i_2) &= 0 \\
-R_2 i_2 - R_4 (i_2 - i_3) - R_6 (i_2 - i_4) - R_3 (i_2 - i_1) &= 0 \\
-V_2 - R_5 i_3 - R_7 (i_3 - i_4) - R_4 (i_3 - i_2) &= 0 \\
V_3 - R_6 (i_4 - i_2) - R_7 (i_4 - i_3) - R_8 i_4 &= 0
\end{align*}
\]

The four equations can be rewritten in matrix form \([A][x] = [B] \):
Chapter 3: Solved Problems

Script file:

clear, clc
V1=28; V2=36; V3=42;
R1=16; R2=10; R3=6; R4=12;
R5=8; R6=14; R7=4; R8=5;
A=[-(R1+R3) R3 0 0
  R3 -(R2+R3+R4+R6) R4 0
  0 R4 -(R4+R5+R7) R7
  0 R6 R7 -(R6+R7+R8)];
B=[-V1; 0; V2; -V3];
I=A\B

Command Window:

I =
  1.2136
-0.2168
-1.3656
  1.4566
>>