General Modelling and Scaling Laws

- Dimensionless numbers
- Similarity requirements
- Derivation of dimensionless numbers used in model testing
- Froude scaling
- Hydroelasticity
- Cavitation number

Chapter 2 in the lecture notes

Dimensionless numbers

- *"Without dimensionless numbers, experimental progress in fluid mechanics would have been almost nil;*
  It would have been swamped by masses of accumulated data" (R. Olson)
- Example:
  Due to the beauty of dimensionless numbers, C_f of a flat, smooth plate is a function of *Re* only
  (not function of temperature, pressure or type of fluid[1])

[1]As long as the fluid is Newtonian, which means that it has a linear stress/strain rate, with zero stress for zero strain

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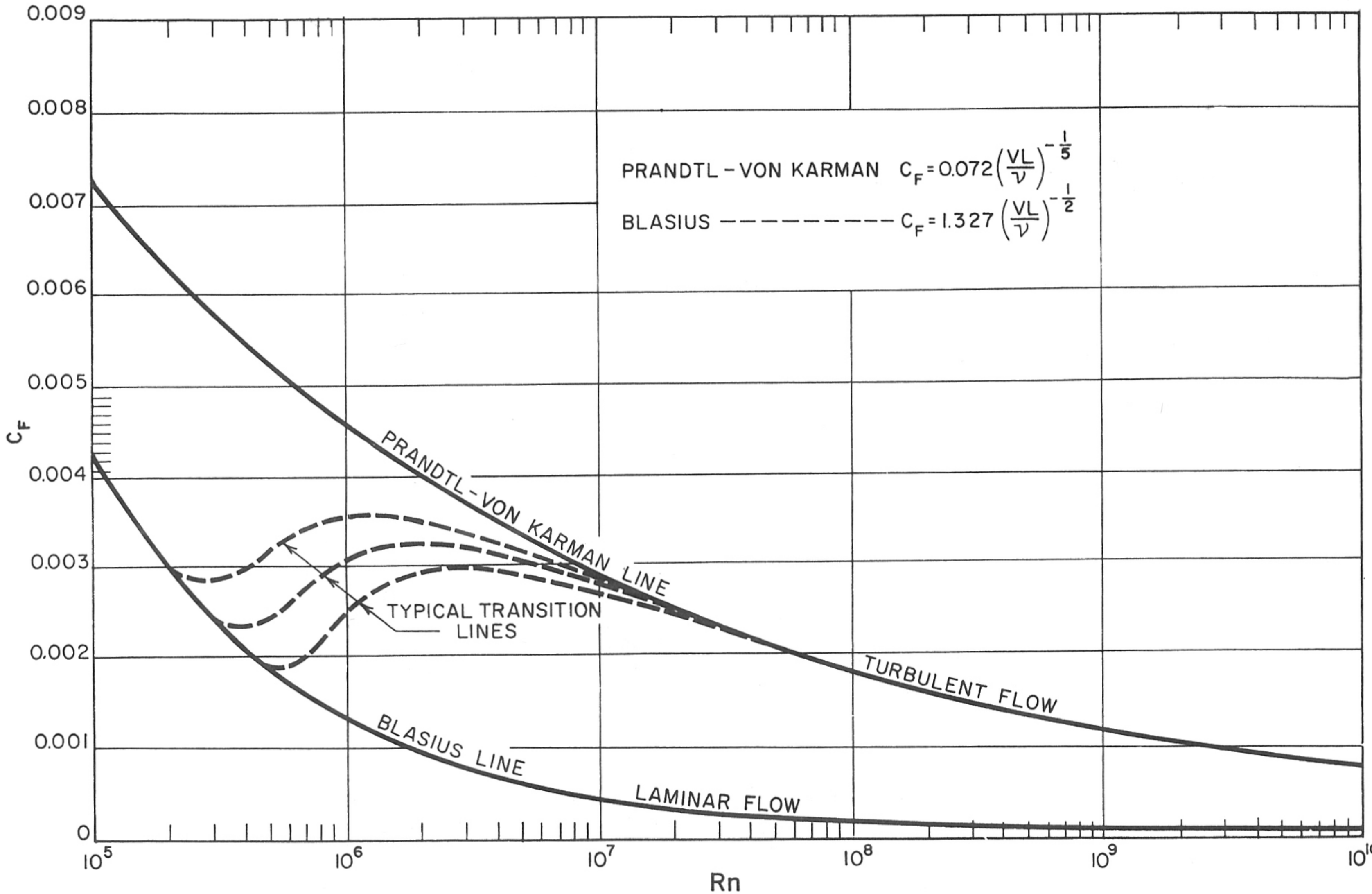

Fig. 2 Skin friction lines, turbulent and laminar flow

Types of similarity

- Geometrical similarity
- Kinematic similarity
- Dynamic similarity

What are the similarity requirements for a model test?

Geometrical Similarity

- The model and full scale structures must have the same shape

$\Rightarrow$ All linear dimensions must have the same scale ratio:

$$\lambda = L_F / L_M$$

- This applies also to:
  - The environment surrounding the model and ship
  - Elastic deformations of the model and ship

Kinematic Similarity

- Similarity of velocities:

$\Rightarrow$ The flow and model(s) will have geometrically similar motions in model and full scale

<u>Examples:</u>

- Velocities in x and y direction must have the same ratio, so that a circular motion in full scale must be a circular motion also in model scale
- The ratio between propeller tip speed and advance speed must be the same in model and full scale:

$$\frac{V_F}{n_F(2\pi R_F)} = \frac{V_M}{n_M(2\pi R_M)} \quad \text{or} \quad \frac{V_F}{n_F D_F} = \frac{V_M}{n_M D_M} \Rightarrow J_F = J_M$$

Dynamic Similarity

- Geometric similarity
  and
- Similarity of forces
  - ⇒ *Ratios between different forces in full scale must be the same in model scale*

⇒ If you have geometric and dynamic similarity, you'll also have kinematic similarity

- The following force contributions are of importance:
  - Inertia Forces, F_i
  - Viscous forces, F_v
  - Gravitational forces, F_g
  - Pressure forces, F_p
  - Elastic forces in the fluid (compressibility), F_e.
  - Surface forces, F_s.

Inertia Forces (mass forces)

$$F_i \propto \rho \frac{dU}{dt} L^3 = \rho \frac{dU}{dx}\frac{dx}{dt} L^3 \propto \rho U^2 L^2$$

- ρ is fluid density
- *U* is a characteristic velocity
- *t* is time
- *L* is a characteristic length (linear dimension)

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Gravitational Forces

$$F_g \propto \rho g L^3$$

$\Rightarrow$ Just mass times acceleration

- g is acceleration of gravity

Viscous Forces

$$F_v \propto \mu \frac{dU}{dx} L^2 \propto \mu U L$$

- μ is dynamic viscosity [kg/m·s]
  - a function of temperature and type of fluid

Pressure Forces

$$F_p \propto pL^2$$

$\Rightarrow$ Force equals pressure times area

- p is pressure

Elastic Fluid Forces

$$F_e \propto \varepsilon_v E_v L^2$$

- ε_v is compression ratio
- E_v is the volume elasticity (or compressibility)
- $\varepsilon_v \cdot E_v$=elasticity modulus K [kg/m·s^2]

Surface Forces

$$F_s \propto \sigma L$$

- σ is the surface tension [kg/s^2]

Froude number *Fn*

- The ratio between inertia and gravity:

$$\frac{Inertia\ \ force}{Gravity\ \ force} = \frac{F_i}{F_g} \propto \frac{\rho U^2 L^2}{\rho g L^3} = \frac{U^2}{gL}$$

- Dynamic similarity requirement between model and full scale:

$$\frac{U_M{}^2}{gL_M} = \frac{U_F{}^2}{gL_F}$$

$$\frac{U_M}{\sqrt{gL_M}} = \frac{U_F}{\sqrt{gL_F}} = Fn$$

- Equality in *Fn* in model and full scale will ensure that <u>gravity forces</u> are correctly scaled
- Surface waves are gravity-driven ⇒ equality in *Fn* will ensure that wave resistance and other wave forces are correctly scaled

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Reynolds number *Re*

- Equal ratio between inertia and viscous forces:

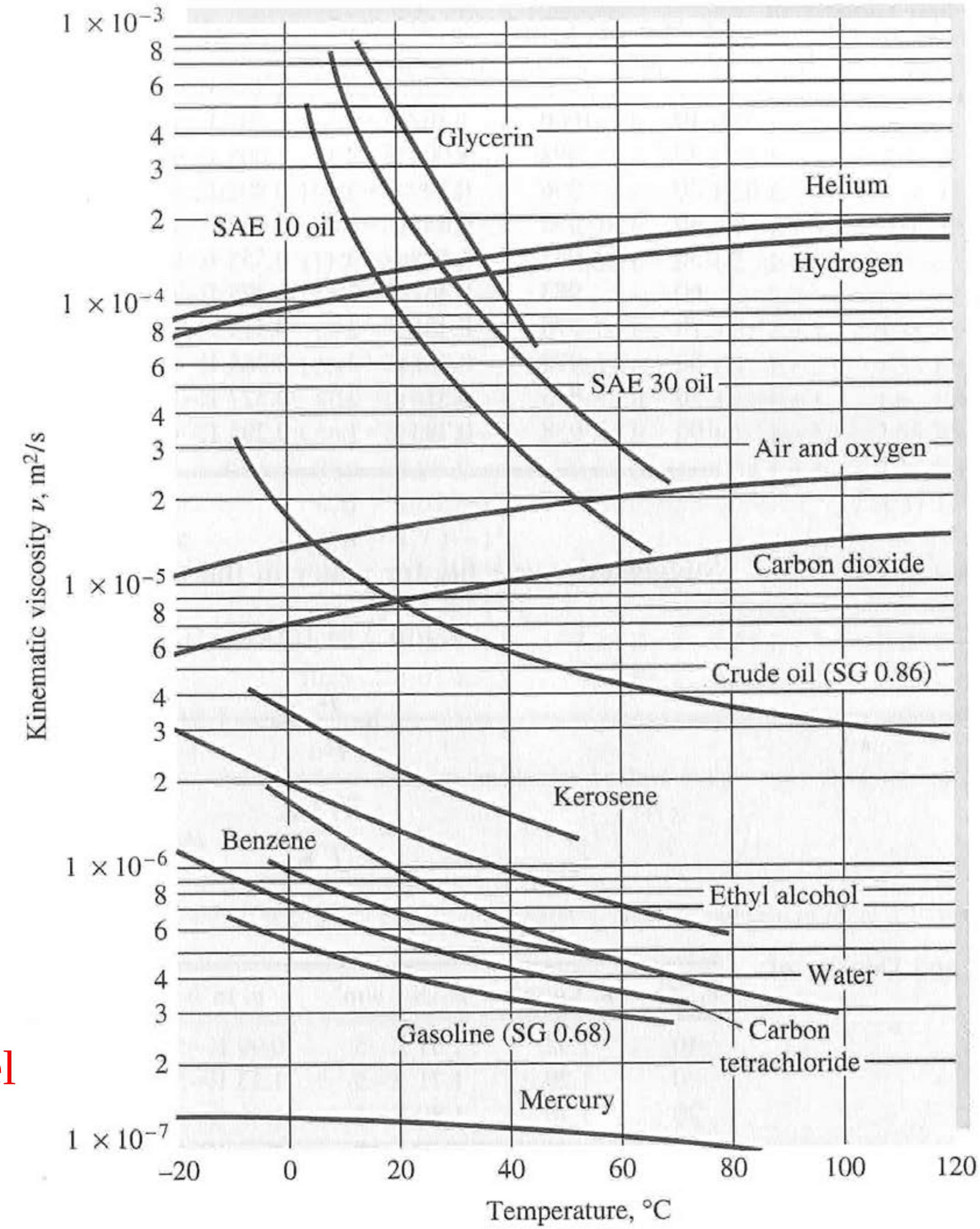
$$\frac{Inertia\ \ forces}{Viscous\ \ forces} = \frac{F_i}{F_v} \propto \frac{\rho U^2 L^2}{\mu UL} = \frac{\rho UL}{\mu} = \frac{UL}{\nu} = Re$$

- ν is the kinematic viscosity, $\nu = \dfrac{\mu}{\rho}$ [m²/s]

- Equality in *Re* will ensure that viscous forces are correctly scaled

Kinematic viscosity of fluids

(from White: Fluid Mechanics)



To obtain equality of both Fn and Rn for a ship model in scale 1:10: $\nu_m = 3.5 \times 10^{-8}$

Mach number M_n

- Equal ratio between inertia and elastic fluid forces:

$$\frac{F_i}{F_e} \propto \frac{\rho U^2 L^2}{\varepsilon_v E_v L^2}$$

- By requiring ε_v to be equal in model and full scale:

$$\left(\frac{\rho U^2 L^2}{\varepsilon_v E_v L^2}\right)_M = \left(\frac{\rho U^2 L^2}{\varepsilon_v E_v L^2}\right)_F$$

$$\frac{U_M}{\sqrt{E_{v,M}/\rho}} = \frac{U_F}{\sqrt{E_{v,F}/\rho}} = M_n$$

- $\sqrt{E_v/\rho}$ is the speed of sound
- Fluid elasticity is very small in water, so usually Mach number similarity is not required
  - It is only when Mach numbers get close to 1 that it is important to consider compressibility effects. When Mach<0.7, incompressible flow is assumed

Weber number W_n

- The ratio between inertia and surface tension forces:

$$\frac{Inertia\ \ forces}{Surface\,tension\ \ forces}=\frac{F_i}{F_s}\propto\frac{\rho U^2L^2}{\sigma L}=\frac{\rho U^2 L}{\sigma}$$

- Similarity requirement for model and full scale forces:

$$\left(\frac{\rho U^2 L}{\sigma}\right)_M=\left(\frac{\rho U^2 L}{\sigma}\right)_F$$

σ=0.073 at 20°C

$$\frac{U_M}{\sqrt{\sigma_M/(\rho L)_M}}=\frac{U_F}{\sqrt{\sigma_F/(\rho L)_F}}=W_n$$

When Wn>180, we assume that a further increase in Wn doesn't influence the fluid forces

Scaling ratios used in testing of ships and offshore structures

Symbol	Dimensionless Number	Force Ratio	Definition
R_e	Reynolds Number	Inertia/Viscous	$\frac{UL}{\nu}$
F_n	Froude Number	Inertia/Gravity	$\frac{U}{\sqrt{gL}}$
M_n	Mach's Number	Inertia/Elasticity	$\frac{U}{\sqrt{E_V/\rho}}$
W_n	Weber's Number	Inertia/Surface tension	$\frac{U}{\sqrt{\sigma/\rho L}}$
St	Strouhall number	-	$\frac{f_v D}{U}$
KC	Keulegan-Carpenter Number	Drag/Inertia	$\frac{U_A T}{D}$

Froude Scaling

$$\frac{U_M}{\sqrt{gL_M}} = \frac{U_F}{\sqrt{gL_F}} \quad \Rightarrow \quad U_F = U_M \sqrt{\frac{L_F}{L_M}} = U_M \sqrt{\lambda}$$

Using the geometrical similarity requirement: $\lambda = L_F / L_M$

If you remember this, most of the other scaling relations can be easily derived just from the physical units

Froude scaling table

Physical Parameter	Unit	Multiplication factor
Length	[m]	λ
Structural mass:	[kg]	$\lambda^3 \cdot \rho_F / \rho_M$
Force:	[N]	$\lambda^3 \cdot \rho_F / \rho_M$
Moment:	[Nm]	$\lambda^4 \cdot \rho_F / \rho_M$
Acceleration:	[m/s^2]	$a_F = a_M$
Time:	[s]	$\sqrt{\lambda}$
Pressure:	[Pa=N/m^2]	$\lambda \cdot \rho_F / \rho_M$

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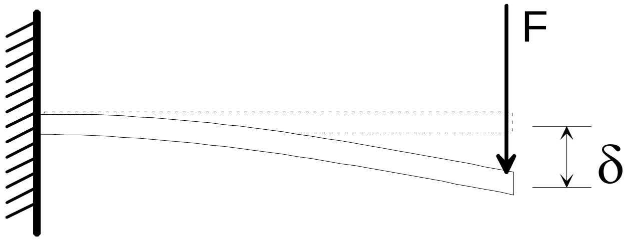
Hydroelasticity

- Additional requirements to the elastic model
  - Correctly scaled global stiffness
  - Structural damping must be similar to full scale
  - The mass distribution must be similar

- Typical applications:
  - Springing and whipping of ships
  - Dynamic behaviour of marine risers and mooring lines

Scaling of elasticity



$$\delta \propto \frac{FL^3}{EI}$$

Hydrodynamic force: $F \propto C\rho U^2 L^2$

Geometric similarity requirement: $\frac{\delta_F}{L_F} = \frac{\delta_M}{L_M} \Rightarrow \delta_F = \lambda \delta_M$

Requirement to structural rigidity:

$$\left(\frac{U^2 L^4}{EI}\right)_F = \left(\frac{U^2 L^4}{EI}\right)_M \Rightarrow (EI)_F = (EI)_M \, \lambda^5$$

Scaling of elasticity – geometrically similar models

- Geometrically similar model implies: $I_F = I_M \lambda^4$
- Must change the elasticity of material: $E_F = E_M \lambda$
- Elastic propellers must be made geometrically similar, using a very soft material: $E_M = E_F / \lambda$
- Elastic hull models are made geometrically similar only on the outside. Thus, E is not scaled and $I_M = I_F \cdot \lambda^{-5}$

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Cavitation

- Dynamic similarity requires that cavitation is modelled
- Cavitation is correctly modelled by equality in cavitation number:

$$\sigma = \frac{(\rho g h + p_0) - p_v}{1/2 \rho U^2}$$

- To obtain equality in cavitation number, atmospheric pressure p_0 might be scaled
- p_v is vapour pressure and $\rho g h$ is hydrostatic pressure
- Different ”definitions” of the velocity U is used

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