and since the depth y varies along the channel, the flow is not fully developed. Thus, the velocity varies along the channel in nonuniform flow.

15.3 THE IMPORTANCE OF THE FROUDE NUMBER

An open channel flow may be steady or unsteady, laminar or turbulent, and it may involve any liquid. For example, if you topple a can of motor oil on a sloped pavement, a sheet of oil will flow downhill and create a laminar, unconfined open channel flow. On the other hand, the flow in a river is a turbulent, confined open channel flow. Either of these flows may be steady or unsteady depending on the circumstances.

The important parameters governing an open channel flow are revealed by a dimensional analysis. The DA of open channel flow begins by recognizing that for a given channel geometry, the important factors are the density and viscosity of the fluid, the speed of the flow V, gravity, a length scale L, and surface tension σ. After a dimensional analysis has been performed, the dimensionless parameters are found to be the Reynolds number $Re = \rho V L/\mu$, the Froude number $Fr = V/\sqrt{gL}$, and the Weber number $We = \rho V^2 L/\sigma$.

Consider first the role of surface tension in open channel flow. The Weber number may be thought of as the ratio of flow kinetic energy $\rho V^2 L^3$ to the surface energy σL^2 associated with the surface tension. With the exception of a thin sheet of liquid flowing downhill, as in the case of spilled motor oil mentioned earlier, the Weber numbers of open channel flows are very large. For example, for water at 60°F flowing at 1 ft/s in a river with a hydraulic radius of 20 ft we find:

$$We = \frac{\rho V^2 R_H}{\sigma} = \frac{(1.938\ \text{slugs/ft}^3)(1.00\ \text{ft/s})^2(20.0\ \text{ft})}{5.03 \times 10^{-3}\ \text{lb}_f/\text{ft}} = 7710$$

Thus the surface energy is negligible in comparison to the flow kinetic energy, and we can safely ignore surface tension effects in large-scale open channel flows.

The value of the Reynolds number is important in predicting whether an open channel flow is laminar or turbulent. In most cases of interest, Re is relatively large and the flow is turbulent. As a rule of thumb, an open channel flow may be assumed to be laminar for Re based on hydraulic radius of less than 500 and turbulent if Re exceeds 12,500. Using the data just given for the river, Re is found to be

$$Re_R = \frac{\rho V R_H}{\mu} = \frac{(1.938\ \text{slugs/ft}^3)(1\ \text{ft/s})(20\ \text{ft})}{2.344 \times 10^{-5}\ (\text{lb}_f\text{-s})/\text{ft}^2} = 1.7 \times 10^6$$

This is well into the turbulent flow range. Observation suggests that a flow in a culvert or flood control channel is also likely to be turbulent. As will be discussed in more detail later, the frictional forces in large Re open channel flows are found to depend primarily on the roughness of the channel walls; they are nearly independent of Re. Thus, Reynolds number is not as important in open channel flow as in other flows you have studied.

The Froude number proves to be the single significant dimensionless parameter in open channel flow. Using the river data given earlier, the Froude number is calculated as

$$Fr = \frac{V}{\sqrt{g R_H}} = \frac{1\ \text{ft/s}}{\sqrt{(32.2\ \text{ft/s}^2)(20\ \text{ft})}} = 0.04$$

As will be explained further in the next section, this value corresponds to what is called subcritical flow. Critical flow occurs at $Fr = 1$, and supercritical flow for $Fr > 1$. The value of the Froude number is implicated in a number of interesting phenomena in open channel flow, including flow over a bump or depression, the behavior of waves on a free surface, the response of the flow to a change in channel area, and the phenomenon known as hydraulic jump. This will be demonstrated in our discussion of these topics in Sections 15.3.1 through 15.3.4. We begin by showing that the response of an open channel flow to a bump or depression in the bed of the channel depends on whether the flow is subcritical or supercritical.

15.3.1 Flow over a Bump or Depression

FE

Consider steady open channel flow in a horizontal rectangular channel of width w that has a bump in the channel bed. To focus solely on the effects of the bump on the flow, we will ignore friction. The geometry of the channel bed and coordinates used to describe the flow over a bump are shown in Figure 15.12A. The liquid depth at any location x along the channel is given by $y(x)$, and the height of the channel bed above an arbitrarily chosen datum elevation is given by $h(x)$. The channel bed and its bump are described by $z = h(x)$, and the free surface is given by $z = y(x) + h(x)$. Upstream of the bump at station x_1 in the horizontal section, the depth of the stream is y_1, the bed height is $z_1 = h(x_1)$, the uniform velocity is V_1, and the flow area is $A_1 = y_1 w$. At a downstream station located at x, the depth of the stream is $y(x)$, the bed height is $z = h(x)$, the uniform velocity is $V(x)$, and the flow area is $A(x) = y(x)w$. Our goal is to predict the depth, knowing the shape of the channel bed and the upstream conditions. Thus the problem is to find the function $y(x)$ with the function $h(x)$ and the upstream flow conditions known.

Applying a steady flow mass balance to the control volume shown in Figure 15.12A, we have $\dot{M} = \rho V_1 A_1 = \rho V(x) A(x)$, which after substituting for the flow areas and dividing by the density gives $Q = V_1 y_1 w = V(x) y(x) w$. Thus we can write

$$V(x) = \frac{V_1 y_1}{y(x)} \tag{15.4}$$

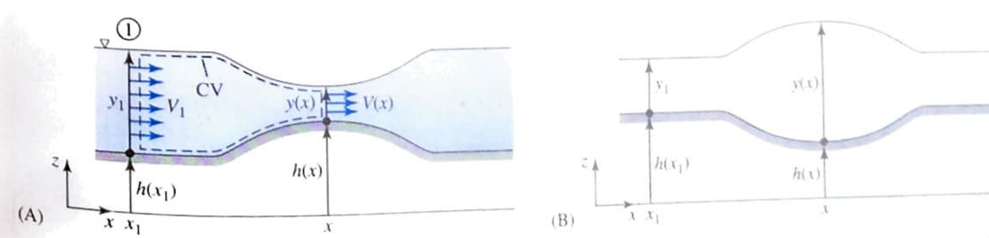

Figure 15.12 (A) Flow over a bump. (B) Flow over a depression.

SOLUTION

We can solve this problem by applying Eq. 15.13 between a point upstream where the velocity, width, and depth are V_1, w_1, and y_1, and a point downstream where the velocity, width, and depth are V_2, w_2, and y_2 as shown in Figure 15.18. Writing Eq. 15.13 between these points gives

$$\frac{1}{2g}\left[\frac{V_1 y_1 w_1}{y_2 w_2}\right]^2 + y_2 = \frac{V_1^2}{2g} + y_1$$

Multiplying by y_2^2 and rearranging we obtain the cubic equation

$$y_2^3 - y_2^2\left[\frac{V_1^2}{2g} + y_1\right] + \frac{1}{2g}\left[\frac{V_1 y_1 w_1}{w_2}\right]^2 = 0$$

After inserting the data, we have $y_2^3 - y_2^2[1.062 \text{ ft}] + 0.1104 \text{ ft} = 0$, which can be solved to obtain the three solutions: -0.286 ft, 0.412 ft, and 0.936 ft. The negative root can be immediately discarded on physical grounds. Next we calculate the downstream velocities and corresponding Froude numbers for each of the positive roots. For $y_2 = 0.412$ ft we find:

$$V_2 = \frac{V_1 y_1 w_1}{y_2 w_2} = \frac{(2 \text{ ft/s})(1 \text{ ft})(4 \text{ ft})}{(0.412 \text{ ft})(3 \text{ ft})} = 6.47 \text{ ft/s}$$

and

$$Fr_2 = \frac{V_2}{\sqrt{g y_1}} = \frac{6.47 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(1 \text{ ft})}} = 1.14$$

For $y_2 = 0.936$ ft we find:

$$V_2 = \frac{V_1 y_1 w_1}{y_2 w_2} = \frac{(2 \text{ ft/s})(1 \text{ ft})(4 \text{ ft})}{(0.936 \text{ ft})(3 \text{ ft})} = 2.85 \text{ ft/s}$$

and

$$F_2 = \frac{V_2}{\sqrt{g y_1}} = \frac{2.85 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(1 \text{ ft})}} = 0.50$$

Calculating the upstream Froude number, we find

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(1 \text{ ft})}} = 0.352$$

hence the flow is subcritical. Since the depth of a subcritical for flow must decrease in a contraction, the correct depth downstream of the contraction is 0.936 ft and the corresponding flow speed is 2.85 ft/s.

It is clear that the behavior of a flow through a channel of varying width is similar in many respects to the flow over a bump or depression. Both involve a change in flow area. If we compare the corresponding equations for the depth slope in flow through an open channel of varying width as given by Eq. 15.14,

$$\frac{dy}{dx} = \frac{(V^2/gw)(dw/dx)}{1 - Fr^2}$$

with the corresponding result for the depth slope in flow over a bump or depression, Eq. 15.9a,

$$\frac{dy}{dx} = \frac{-dh/dx}{1 - Fr^2}$$

we see that a contraction ($dw/dx < 0$) is similar in its effect on depth slope to a bump ($dh/dx > 0$), and an expansion ($dw/dx > 0$) is similar in effect to a depression ($dh/dx < 0$). In both types of flow, knowing the value of the Froude number allows us to make a qualitative prediction of the change in depth and behavior of the free surface.

15.3.3 Propagation of Surface Waves

The behavior of surface waves in open channel flow is also governed by the value of the local Froude number. We can demonstrate this important result by considering a horizontal rectangular channel of width, w, filled with liquid at rest to a uniform depth, y. A vertical wall at the left end of this channel is suddenly given a small constant velocity $V_W\mathbf{i}$ to the right, creating a surface wave of height Δy that propagates down the channel as shown in Figure 15.19A. Note that the depth of the liquid behind the wave is $y + \Delta y$ and that the wave propagates at a constant speed c. The fluid ahead of the wave is at rest, and the fluid behind the wave must be all moving to the right at speed V_W as shown in Figure 15.19A. If you are wondering about this last statement, think about how the moving wall is pushing the liquid to the right. Since the liquid cannot be compressed, and the depth behind the wave is uniform, the liquid between the wall and the wave front must be moving to the right at the same speed as the wall.

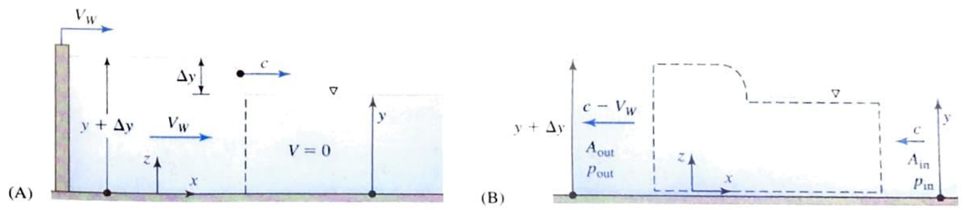

Figure 15.19 The moving end wall with (A) a fixed reference frame and (B) a moving reference frame.

We wish to analyze this model of wave motion and predict the relationships between wave speed c, wave height Δy, and the other physical parameters of the problem. Since the wave is propagating along the channel, the flow is unsteady for an observer in a fixed reference frame attached to the channel. Rather than analyze this unsteady flow, we can use a reference frame that is moving with the wave as shown in Figure 15.19B. Since this reference frame is moving at constant velocity, it is inertial. In the moving frame liquid appears to be approaching the stationary wave from the right at speed c, and moving at speed $c - V_W$ to the left after passing through the wave. Thus the uniform velocity vector on the upstream side of the stationary wave is $\mathbf{u} = -c\mathbf{i}$ and on the downstream side it is $\mathbf{u} = -(c - V_W)\mathbf{i}$ as shown in Figure 15.19B. The flow is steady in this frame.

Applying a steady flow mass balance to the control volume shown in Figure 15.19B, we obtain $-\rho c A_{in} + \rho(c - V_W)A_{out} = 0$, where the inlet and outlet flow areas can be seen to be given by $A_{in} = wy$ and $A_{out} = w(y + \Delta y)$. Thus the mass balance can be written as

$$-\rho c y + \rho(c - V_W)(y + \Delta y) = 0 \tag{15.15}$$

and we can solve for the wave speed to find

$$c = \frac{V_W(y + \Delta y)}{\Delta y} \tag{15.16}$$

At this point, the wave height Δy is unknown, so this single equation is not sufficient to determine the wave speed. Alternately we can use Eq. 15.16 to write

$$V_W = \frac{c\Delta y}{y + \Delta y} \tag{15.17}$$

We can obtain a second equation by applying a steady flow momentum balance in the x direction to the control volume shown in Figure 15.19B. We will neglect friction on the bed surface and assume that the pressure distributions on the inlet and exit surfaces are hydrostatic. The resulting surface forces can then be calculated to be $-wy(p_A + \rho g y/2)\mathbf{i}$ on the inlet flow area, $w(y + \Delta y)\{p_A + [\rho g(y + \Delta y)]/2\}\mathbf{i}$ on the outlet flow area, and a contribution $-p_A w \Delta y \mathbf{i}$ from the free surface at atmospheric pressure. The momentum balance in the x direction is

$$\rho c^2 wy - \rho(c - V_W)^2 w(y + \Delta y)$$
$$= -wy\left[p_A + \frac{\rho g y}{2}\right] + w(y + \Delta y)\left[p_A + \frac{\rho g(y + \Delta y)}{2}\right] - p_A w \Delta y$$

We can write the flux term at the outlet as

$$-\rho(c - V_W)^2 w(y + \Delta y) = -\rho w(c - V_W)[(c - V_W)(y + \Delta y)]$$

then use the mass balance, Eq. 15.15, to replace the term in the square brackets by cy getting

$$-\rho(c - V_W)^2 w(y + \Delta y) = -\rho w c y(c - V_W)$$

Thus the two flux terms give $\rho c^2 wy - \rho w c y(c - V_W) = \rho w c y V_W$. Adding and simplifying the three surface force terms, we obtain $w\Delta y(\rho g y + \rho g \Delta y/2)$; thus the momentum balance is

$$\rho w c y V_W = w\Delta y\left[\rho g y + \frac{\rho g \Delta y}{2}\right]$$

Solving for the wave speed we have

$$c = \frac{g\Delta y}{V_W}\left[1 + \frac{\Delta y}{2y}\right] \tag{15.18}$$

Equations 15.16 and 15.18 can now be used to determine the wave speed c and wave height Δy that describe the wave produced when a wall moves at velocity V_W.

The moving wall concept was introduced at the beginning of our analysis as simply the easiest way to visualize a wave propagating along a free surface. At this point we can dispense with the moving wall. That is, we can now simply imagine a solitary wave propagating at some wave speed c, with a wave height Δy, with the liquid behind the wave moving in the same direction at speed V_W. The preceding analysis holds for this isolated wave. Thus, using Eq. 15.17: $V_W = c\Delta y/(y + \Delta y)$, to eliminate V_W from Eq. 15.18, we find that the wave speed obeys the equation:

$$c = \sqrt{gy\left[1 + \frac{\Delta y}{2y}\right]\left[1 + \frac{\Delta y}{y}\right]} \tag{15.19}$$

This equation applies to a wave moving in either direction. Equation 15.19 shows that wave speed increases as the wave height, Δy, increases. Also, by Eq. 15.17, a wave traveling at speed c in stationary water causes the water to move at a speed $V_W = c\Delta y/(y + \Delta y)$ in the direction of wave propagation.

If the amplitude of a wave is sufficiently small, i.e., $\Delta y/y \ll 1$, Eq. 15.19 simplifies to

$$c = \sqrt{gy} \tag{15.20}$$

We see that a small amplitude wave travels at a speed that is determined by the water depth in which it is propagating. In water of uniform depth, a small amplitude wave propagates at a constant speed, and since the wave speed does not depend on Δy, the wave travels without change of shape. Introducing the small amplitude approximation $\Delta y/y \ll 1$ into Eq. 15.17 shows that such a wave causes a velocity in the direction of propagation of $V_W = c\Delta y/y$. The wave created in a still pond by tossing a rock into the water is likely to satisfy the small amplitude assumption. Can you think of a way of determining the water depth in pond or lake by observing the propagation of ripples?

We can also analyze a wave propagating on the surface of a moving stream, i.e., a wave traveling in an open channel flow. Consider first a wave traveling upstream at speed c in a stream moving at speed V. This wave can be analyzed by making use of a change

The speed of a finite amplitude wave is always greater than $c = \sqrt{gy}$. This is easily seen by comparing Eqs. 15.19 and 15.20. If you were using $c = \sqrt{gy}$ to estimate the time of arrival of a large amplitude tidal wave moving in shallow water, you might not have allowed yourself enough time to escape.

EXAMPLE 15.4

Figure 15.20 represents a wave-making machine at a water park. The machine works by moving the wall into a pool 1.25 m deep. The wave amplitude is 0.3 m. Find the wave speed and velocity of the water behind the wave.

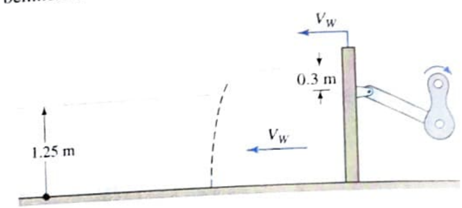

Figure 15.20 Schematic for Example 15.4.

SOLUTION

The wave speed is given by Eq. 15.19 as

$$c = \sqrt{gy\left[1+\frac{\Delta y}{2y}\right]\left[1+\frac{\Delta y}{y}\right]}$$

Inserting the data we obtain

$$c = \sqrt{(9.81\ \text{m/s}^2)(1.25\ \text{m})[1+0.3\ \text{m}/2(1.25\ \text{m})][1+0.3\ \text{m}/1.25\ \text{m}]} = 4.13\ \text{m/s}$$

If we had incorrectly assumed that the wave is of small amplitude and used Eq. 15.20 ($c = \sqrt{gy}$), we would have obtained $c = \sqrt{gy} = \sqrt{(9.8\ \text{m/s}^2)1.25\ \text{m}} = 3.5\ \text{m/s}$. By checking the ratio of wave amplitude to depth, i.e., $\Delta y/y = 0.3\ \text{m}/1.25\ \text{m} = 0.24$, we would have recognized that the small amplitude approximation is invalid and likely to produce erroneous results.

The speed of the water behind the wave, which is the same as the wall speed, can be found by using Eq. 15.17 and the correct wave speed $c = 4.13$ m/s. The result is

$$V_W = \frac{c\Delta y}{y+\Delta y} = \frac{(4.13\ \text{m/s})(0.3\ \text{m})}{1.25\ \text{m}+0.3\ \text{m}} = 0.80\ \text{m/s}$$

EXAMPLE 15.5

Most of the waves we observe are generated by the action of the wind over the surface of the water. Tsunamis are waves generated by seismic activity such as earthquakes and volcanoes, or by catastrophic events such as asteroids impacting in the ocean. A tsunami would seem quite harmless if observed on the open ocean, where its amplitude might be as small as 10 cm. However these waves transmit a tremendous amount of energy. What

is the wave speed of a tsunami traveling across the Pacific, where a typical depth is 4000 m? Compare this with the speed of a wave of the same amplitude in 1 m of water.

SOLUTION

The wave is sketched in Figure 15.21A. Clearly the small amplitude approximation, Eq. 15.20, is appropriate for the tsunami in the open ocean. Inserting the data, we have

$$c = \sqrt{gy} = \sqrt{(9.81\ \text{m/s}^2)(4000\ \text{m})} = 198\ \text{m/s}$$

which is over 400 mph! When this wave approaches a coastline, it will slow down because the water is shallower, but its amplitude will grow because the energy flux, which is a function of the speed and amplitude, is constant. Tsunamis are very destructive because their amplitude can easily exceed several meters. Figure 15.21B shows the effect of a tsunami that struck Hawaii in 1960.

For a 10 cm amplitude wave in 1 m deep water, we can calculate the wave speed from

$$c = \sqrt{gy\left[1+\frac{\Delta y}{2y}\right]\left[1+\frac{\Delta y}{y}\right]} = \sqrt{(9.81\ \text{m/s}^2)(1\ \text{m})\left[1+\frac{0.1\ \text{m}}{2(1\ \text{m})}\right]\left[1+\frac{0.1\ \text{m}}{1\ \text{m}}\right]}$$
$$= 3.37\ \text{m/s}$$

Using the small amplitude approximation $c = \sqrt{gy} = \sqrt{(9.81\ \text{m/s}^2)(1\ \text{m})}$ for this wave yields a wave speed of 3.13 m/s, which is in error by only about 7%.

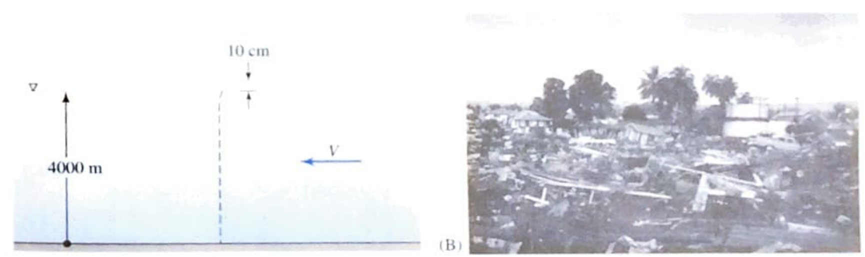

Figure 15.21 (A) Schematic for Example 15.5. (B) Damage due to a tsunami that struck Hilo, Hawaii, in 1960.

of reference frame. Figure 15.22A shows the wave as seen by an observer fixed as usual in the frame of reference attached to the channel. In Figure 15.22B, however, we have changed to a frame of reference fixed to the undisturbed stream moving at speed V. In this moving reference frame, the wave speed is now $c + V$, and the water behind the wave is moving in the direction of wave motion at speed $V - V_R$. The wave now looks exactly like the preceding case of a wave propagating into stationary water, provided we write $V_W = V - V_R$ and recognize that the wave speed in the stationary water case is such as wavelength may come into play in determining wave speed. For example, as you have seen at the beach, waves break when they reach shallow water. Nevertheless, we once again see that the value of the Froude number is of paramount importance in predicting the behavior of isolated surface waves and understanding the flow of information in open channel flow.

The hydraulic jump discussed in the next section can be thought of as a special case of a surface wave. However, its importance in open channel flow merits a separate treatment and discussion. As you will learn, a hydraulic jump can occur only when the flow is supercritical, that is, when the Froude number is greater than one.

15.3.4 Hydraulic Jump

In Section 15.3.3 we showed that if a flow is supercritical, disturbances cannot propagate upstream. Thus if downstream conditions require that a supercritical flow become subcritical, a smooth transition is impossible, and the flow goes through a phenomenon known as a hydraulic jump. In a hydraulic jump, the flow changes from supercritical to subcritical in a relatively short distance, with an abrupt decrease in velocity, increase in depth, and a substantial head loss. An example of a hydraulic jump on a dam spillway was shown in Figure 15.3, but you can also create a hydraulic jump yourself by running water from the kitchen faucet onto a flat or slightly sloped surface. As the flow moves radially outward on the surface, a hydraulic jump will occur if the flowrate is large enough.

The general characteristics of a hydraulic jump are shown in Figure 15.24. A hydraulic jump may occur on an inclined or horizontal bed, and in a channel of any shape. For simplicity, we will assume a rectangular horizontal channel. Observations have determined that the maximum length of a jump does not exceed seven downstream depths, thus we will neglect the shear stress applied to the flow by the bed. Note, however, that we are not assuming frictionless flow. In fact, our analysis must allow for a head loss due to viscous dissipation in the jump. Although the turbulent velocity field in a hydraulic jump is generally 3D, we can arrive at the important characteristics by assuming steady, uniform flow, and applying a mass, momentum, and energy balance to the control volume as shown in Figure 15.24.

A mass balance on this CV yields $\dot{M} = \rho Q = \rho A_1 V_1 = \rho A_2 V_2$, where $A_1 = wy_1$ and $A_2 = wy_2$. Thus we can write

$$V_1 y_1 = V_2 y_2 \tag{15.24}$$

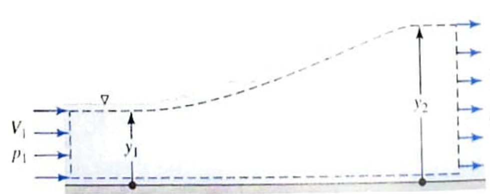

Figure 15.24 Control volume for the hydraulic jump.

To apply a momentum balance, we use Eq. 7.19b:

$$\int_{CS} (\rho \mathbf{u})(\mathbf{u}\cdot\mathbf{n})\,dS = \int_{CV} \rho \mathbf{f}\,dV + \int_{CS} \Sigma\,dS$$

and consider the component of this equation in the x (flow) direction. The flux terms on the inlet and outlet give $[-\rho V_1^2 A_1 + \rho V_2^2 A_2]\mathbf{i}$, which we can write as $[-\rho V_1^2 wy_1 + \rho V_2^2 wy_2]\mathbf{i}$. There is no component of the body force in the x direction. To evaluate the stress term, we will assume that the pressure distribution on the inlet and exit are hydrostatic and given by $p(y) = p_A - \rho g(y - y_1)$ and $p(y) = p_A - \rho g(y - y_2)$, respectively. The pressure on the free surface is atmospheric, and we can account for the net force applied on this surface in the x direction by using gage pressure in the inlet and outlet terms. On the bottom, the hydrostatic pressure varies from inlet to outlet but contributes no net force in the x direction. The effect of the shear stress here is neglected as explained earlier. The surface force terms involving pressure are

$$\int_{\text{inlet}} -p\mathbf{n}\,dS + \int_{\text{outlet}} -p\mathbf{n}\,dS = \int_0^{y_1} -(-\rho g(y - y_1))(-\mathbf{i})w\,dy + \int_0^{y_2} -(-\rho g(y - y_2))(\mathbf{i})w\,dy$$

which yield $[\rho g w(y_1^2/2) - \rho g w(y_2^2/2)]\mathbf{i}$. Combining terms, we see that the momentum balance yields $-\rho V_1^2 wy_1 + \rho V_2^2 wy_2 = \rho g w(y_1^2/2) - \rho g w(y_2^2/2)$. Our final result after rearranging is

$$\frac{V_1^2}{g} y_1 + \frac{y_1^2}{2} = \frac{V_2^2}{g} y_2 + \frac{y_2^2}{2} \tag{15.25}$$

We can write an energy balance for this CV by employing Eq. 7.33:

$$\int_{CV} \frac{\partial}{\partial t}\left(\rho\left(u + \frac{1}{2}\mathbf{u}\cdot\mathbf{u} + gz\right)\right) dV + \int_{CS} \rho\left(u + \frac{p}{\rho} + \frac{1}{2}\mathbf{u}\cdot\mathbf{u} + gz\right)(\mathbf{u}\cdot\mathbf{n})\,dS$$
$$= \dot{W}_{\text{power}} + \dot{W}_{\text{shaft}} + \dot{Q}_C + \dot{S}$$

There is no fluid power, shaft power, or other energy input in this case, and the flow is steady. Thus after the inlet and outlet surfaces of the CV have been identified, the energy balance is given by

$$\int_{\text{inlet}} \rho\left(u + \frac{p}{\rho} + \frac{1}{2}\mathbf{u}\cdot\mathbf{u} + gz\right)(\mathbf{u}\cdot\mathbf{n})\,dS + \int_{\text{outlet}} \rho\left(u + \frac{p}{\rho} + \frac{1}{2}\mathbf{u}\cdot\mathbf{u} + gz\right)(\mathbf{u}\cdot\mathbf{n})\,dS = \dot{Q}_C$$

On the inlet surface, the streamwise velocity is V_1, the uniform internal energy is u_1, the pressure is $p(y) = p_A - \rho g(y - y_1)$, and the gravitational potential energy is given by $gz = gy$. Thus on the inlet we can write

$$\frac{p}{\rho} + gz = \frac{p_A}{\rho} - g(y - y_1) + gy = \frac{p_A}{\rho} + gy_1$$