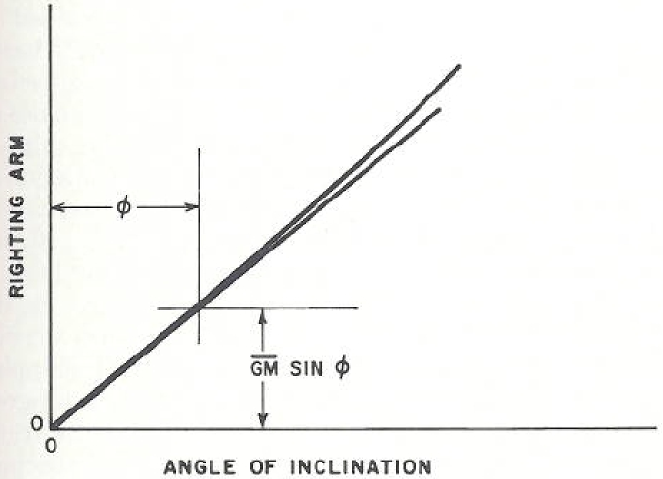

Fig. 45

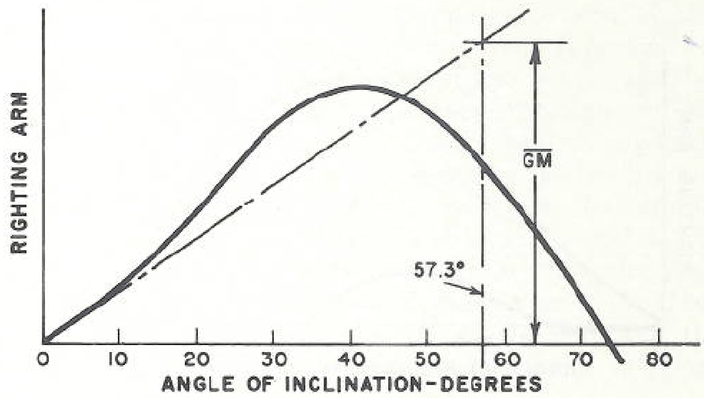

Fig. 46 Slope of stability curve at the origin

ship's center of gravity caused by the reduction in low weight.

When the lines are made finer in this manner, the metacentric height is increased by the difference in the rise of the center of buoyancy and the rise of the center of gravity. An additional effect on metacentric height may be produced by a change in the moment of inertia of the waterplane as the ship settles to the deeper waterline.

**7.5 Significance of the Statical Stability Curve.** The statical stability curve has a number of features which are significant in the analysis of the ship's stability.

Where the ship's center of gravity is not on the centerline as in the case illustrated in Fig. 39, the point at which the curve crosses the horizontal axis corresponds to the static angle of heel at which the ship will come to rest in still water.

The slope of the curve at zero degrees is the metacentric height. As discussed in Section 4, the righting arm for small angles of heel may be expressed by the formula

$$\overline{GZ} = \overline{GM} \sin \phi$$

The slope of the curve at the origin, as shown in Fig. 45, is therefore $\overline{GM} \sin \phi/\phi$ or, since $\sin \phi$ approaches $\phi$ as $\phi$ approaches zero, the slope is the metacentric height. If the righting arm continued to increase at the same rate as at the origin it would be equal to $\overline{GM}$ at an inclination of 1 radian, or 57.3 deg, as illustrated in Fig. 46. Therefore, if the value of $\overline{GM}$ is plotted as an ordinate at 57.3 deg, a line connecting the plotted point with the origin would be tangent to the statical stability curve at the origin. This is a convenient check for major error in the initial portion of the righting-arm curve.

In cases where there is considerable free surface in wide, shallow tanks, and the moment of transference of the liquid is used to modify the righting arms, the metacentric height is not calculated, and is therefore not available for use in checking the curve. In such cases, however, the process may be reversed, and the righting-arm curve may be used to determine the effective metacentric height by determining the slope of the curve at the origin. This procedure might be useful if there is occasion to estimate the period of roll from the metacentric height, as discussed in Section 4. The slope at the origin, or the effective metacentric height, is equal to the righting arm at 10 deg divided by 10 deg in radians or 10°/57.3°, which is equivalent to multiplying the righting arm at 10 deg by 5.73.

The area under any portion of a curve of righting moment, such as the shaded area in Fig. 47, represents the work required to heel the ship from angle $A$ to angle $B$. A moment, multiplied by the angle through which it is exerted, represents work. In the case of a ship, where the moment varies with the angle, if $M$ is the moment at any angle of heel, $\phi$, then the work required to rotate the ship against this moment through an angle $d\phi$ is $M\,d\phi$, and the work required to rotate it from $A$ to $B$ is

$$\text{Work} = \int_A^B M\,d\phi$$

which is the area under the curve between $A$ and $B$.

If a ship rolls from $B$ to $A$ in Fig. 47 in still water under the influence of the righting moment, the shaded area, minus any energy expended in overcoming the resistance of the water to the rolling motion, represents the kinetic energy imparted to the ship, which will exist in the form of angular velocity at point $A$. If, in rolling freely in still water, the ship passes through the upright position with a certain angular velocity, which may be translated into a corresponding amount of kinetic energy, it will roll

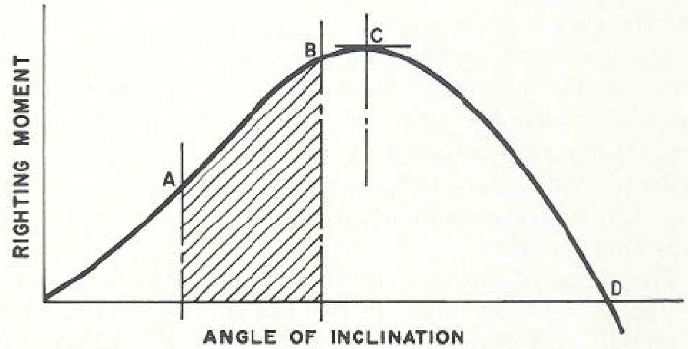

Fig. 47

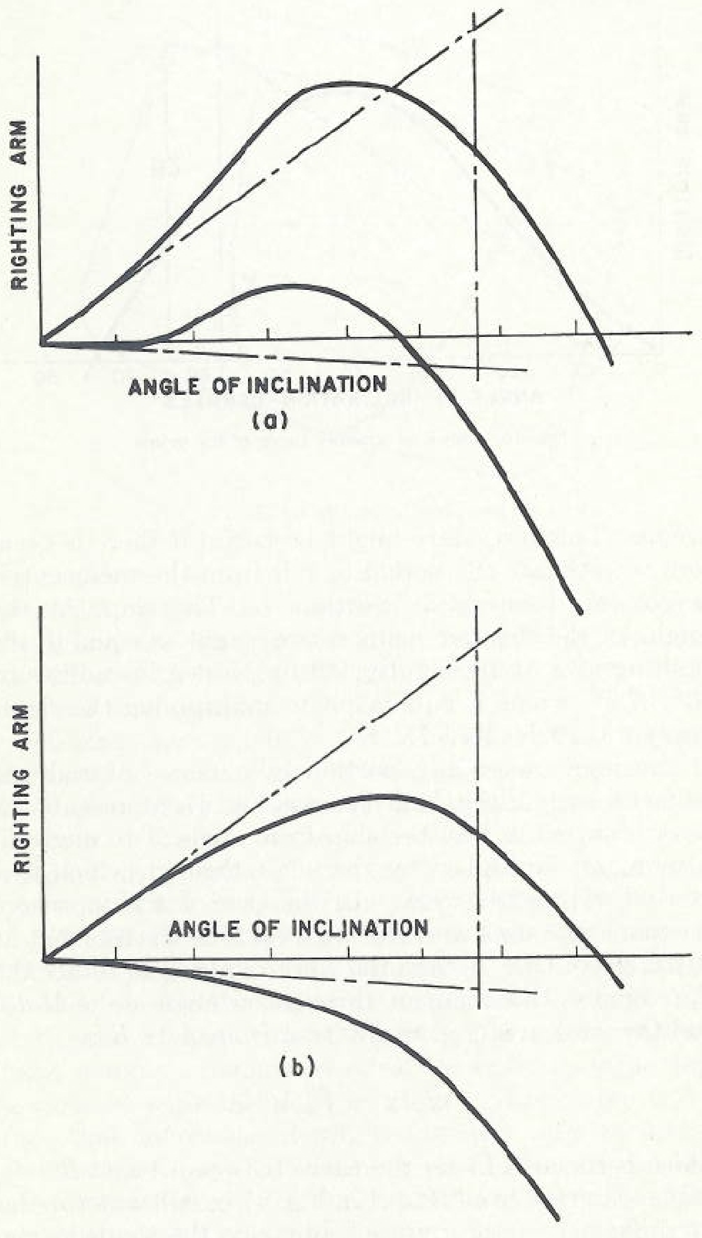


Fig. 48

to an angle such that the area under the righting-moment curve, from the upright to that angle, is equal to the ship's kinetic energy at zero inclination minus the energy absorbed by the resistance of the water. The total area between the righting-moment curve and the horizontal axis represents the total work which is required to capsize the ship from the upright position.

The maximum righting moment, which occurs at angle $C$ in Fig. 47, determines the maximum external upsetting moment which the ship can withstand without capsizing. If the ship is forced over to angle $C$ by an external moment which does not thereafter diminish faster than does the righting moment, it will continue to heel until capsizing occurs.

The range of positive stability is indicated by point $D$ in Fig. 47. If the ship rolls beyond this angle, the forces of weight and buoyancy will act to capsize, rather than to right, the ship. On a normal ship, the range of positive stability is somewhat indefinite. As discussed in Section 3, the cross curves of stability are usually based on the assumption that the superstructure is not effective, and it was pointed out that, at very large angles of heel, there is a possibility that water may be shipped through topside openings with a consequent reduction in stability. If point $D$ is determined from the cross curves as they are customarily calculated, positive righting arms would probably exist if the ship were to roll beyond angle $D$ for a brief period because of the effect of the superstructure. If, on the other hand, the ship were to roll repeatedly to angles approaching point $D$, shipping water through topside openings might cause a progressive reduction in stability which could eventually result in negative righting arms before point $D$ is reached. Shifting of cargo can have a similar result.

The direction of curvature of the statical stability curve near the origin determines whether the ship will develop positive righting arms when the metacentric height is reduced to zero or becomes slightly negative. Two statical stability curves are shown in Fig. 48, for two ships having the same metacentric height but different forms. In Fig. 48($a$), which is typical of cargo and passenger types, the curve is concave upward, while the curve in Fig. 48($b$) is concave downward. Assume that the center of gravity of each ship is shifted upward the same distance, so that the metacentric height in each case becomes slightly negative. At any angle, the righting arm will be reduced by the same amount in each case, the reduction being the product of the vertical movement of the center of gravity and the sine of the angle of inclination. There is an important difference between the resulting statical stability curves; in case ($a$) the ship will heel to a small angle beyond which there will be some positive righting arm, while in case ($b$) the ship will capsize. The condition of negative metacentric height shown in case ($a$) may be recognized by the behavior of the ship, since there will be a list with no apparent heeling moment; the ship comes to rest with a small angle of heel either to port or to starboard but will not remain upright. It is possible for a condition of negative metacentric height to develop gradually in normal operation owing to consuming or unloading low weight, to developing a large free surface or to accumulating topside ice. In such situations, if the righting-arm curve is of the type shown in Fig. 48($a$), and if the existence of negative metacentric height is recognized, there will be some warning that a precarious situation is developing. With a curve of the type shown in Fig. 48($b$), the only warning prior to capsizing would be a lengthening of the period of roll, which would not be apparent if the ship were in still water.

**7.6 Representation of Heeling Moments.** In Section 1, the nature of certain heeling forces was discussed. If the heeling moments developed by these forces are calculated for several angles of inclination, these moments may be plotted on the same coordinates as the statical stability curve, as illustrated in Fig. 49. If the curve labeled "heeling moment" represents the moment of a beam